STATISTICS:
a series of TEXTBOOKS and MONOGRAPHS

Analytical Methods for Risk Management

A Systems Engineering Perspective

Paul R. Garvey

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

Analytical Methods for Risk Management

A Systems Engineering Perspective

STATISTICS: Textbooks and Monographs

Recent Titles

Visualizing Statistical Models and Concepts, *R. W. Farebrother and Michaël Schyns*

Financial and Actuarial Statistics: An Introduction, *Dale S. Borowiak*

Nonparametric Statistical Inference, Fourth Edition, Revised and Expanded, *Jean Dickinson Gibbons and Subhabrata Chakraborti*

Computer-Aided Econometrics, *edited by David E.A. Giles*

The EM Algorithm and Related Statistical Models, *edited by Michiko Watanabe and Kazunori Yamaguchi*

Multivariate Statistical Analysis, Second Edition, Revised and Expanded, *Narayan C. Giri*

Computational Methods in Statistics and Econometrics, *Hisashi Tanizaki*

Applied Sequential Methodologies: Real-World Examples with Data Analysis, *edited by Nitis Mukhopadhyay, Sujay Datta, and Saibal Chattopadhyay*

Handbook of Beta Distribution and Its Applications, *edited by Arjun K. Gupta and Saralees Nadarajah*

Item Response Theory: Parameter Estimation Techniques, Second Edition, *edited by Frank B. Baker and Seock-Ho Kim*

Statistical Methods in Computer Security, *edited by William W. S. Chen*

Elementary Statistical Quality Control, Second Edition, *John T. Burr*

Data Analysis of Asymmetric Structures, *Takayuki Saito and Hiroshi Yadohisa*

Mathematical Statistics with Applications, *Asha Seth Kapadia, Wenyaw Chan, and Lemuel Moyé*

Advances on Models, Characterizations and Applications, *N. Balakrishnan, I. G. Bairamov, and O. L. Gebizlioglu*

Survey Sampling: Theory and Methods, Second Edition, *Arijit Chaudhuri and Horst Stenger*

Statistical Design of Experiments with Engineering Applications, *Kamel Rekab and Muzaffar Shaikh*

Quality by Experimental Design, Third Edition, *Thomas B. Barker*

Handbook of Parallel Computing and Statistics, *Erricos John Kontoghiorghes*

Statistical Inference Based on Divergence Measures, *Leandro Pardo*

A Kalman Filter Primer, *Randy Eubank*

Introductory Statistical Inference, *Nitis Mukhopadhyay*

Handbook of Statistical Distributions with Applications, *K. Krishnamoorthy*

A Course on Queueing Models, *Joti Lal Jain, Sri Gopal Mohanty, and Walter Böhm*

Univariate and Multivariate General Linear Models: Theory and Applications with SAS, Second Edition, *Kevin Kim and Neil Timm*

Randomization Tests, Fourth Edition, *Eugene S. Edgington and Patrick Onghena*

Design and Analysis of Experiments: Classical and Regression Approaches with SAS, *Leonard C. Onyiah*

Analytical Methods for Risk Management: A Systems Engineering Perspective, *Paul R. Garvey*

Confidence Intervals in Generalized Regression Models, *Esa Uusipaikka*

Analytical Methods for Risk Management

A Systems Engineering Perspective

Paul R. Garvey

The MITRE Corporation
Bedford, Massachusetts, U.S.A.

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A CHAPMAN & HALL BOOK

Approved for Public Release (07-0893). Distribution unlimited.

Chapman & Hall/CRC
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-13: 978-1-58488-637-2 (Hardcover)

Library of Congress Cataloging-in-Publication Data

Garvey, Paul R., date.
Analytical methods for risk management : a systems engineering perspective / Paul R. Garvey.
p. cm. -- (Statistics : textbooks and monographs)
Includes bibliographical references and index.
ISBN 978-1-58488-637-2 (alk. paper)
1. Technology--Risk assessment. 2. Risk management. I. Title. II. Series.

T174.5G36 2008
620'.004--dc22 2008012400

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

Dedication

To my wife Maura and daughters Kirsten and Alana.

I also dedicate this book to the memory of my daughter Katrina and my parents Eva and Ralph.

Ad Majorem Dei Gloriam

Contents

Preface

Risk is a driving consideration in decisions that determine how engineering systems are developed, produced, and sustained. Critical to these decisions is an understanding of risk and how it affects the engineering of systems. The process of identifying, measuring, and managing risk is known as risk management. Applied early, risk management can expose potentially crippling areas of risk in the engineering of systems. This provides management time to define and implement corrective strategies. Moreover, risk management can bring realism to technical and managerial decisions that define a system's overall engineering strategy.

Engineering today's systems is sophisticated and complex. Increasingly, systems are being engineered by bringing together many separate systems that, as a whole, provide an overall capability otherwise not possible. Many systems no longer physically exist within clearly defined boundaries; rather, systems are more and more geographically and spatially distributed and interconnected through a rich and sophisticated set of networks and communications technologies.

These large-scale, complex, systems-of-systems operate to satisfy a comparatively large set of users, stakeholders, or communities of interest. It is no longer enough to find just technology solutions to the engineering of these systems. Such solutions must be adaptable to changes in the enterprise, balanced with respect to expected performance, and risk managed, while also considering the social, political, and economic environments within which the system will operate and change over time.

Successfully engineering today's systems requires deliberate and continuous attention to the management of risk. Managing risk is an activity designed to improve the chance that these systems will be completed on time, within cost, and meet performance and capability objectives.

This book presents an introduction to processes and analytical practices in the management of risk as it arises in the engineering of systems. Traditional systems, systems-of-systems, and enterprise systems are considered. These practices have evolved from a variety of systems engineering projects and enterprise engineering initiatives. Numerous examples are presented to illustrate how these principles and practices have been applied on actual engineering system projects and enterprise modernization programs.

A large body of literature exists on this subject. Published material appears in numerous industry and government technical reports, symposia proceedings, and professional society publications. Despite this, engineering managers need to properly understand this literature, learn which processes and analytical techniques are valid, and know how they are best applied in their specific system environments. This book addresses these needs. It provides managers and systems engineers a guide through the foundation processes, analytical principles, and implementation practices of engineering risk management.

This book is appropriate for upper-level undergraduate or graduate students in systems engineering, program management, or engineering management courses of study. As a text, it could be used for a course or elective on engineering risk management. Readers should have a background in project management and systems engineering principles. Additionally, a mathematical background in differential and integral calculus is recommended. Important concepts from probability theory and elementary decision theory, as they apply to engineering risk management, are developed as needed. The book contains 60 exercises to further a reader's understanding of risk management theory and practice.

Chapter 1 presents an introduction to engineering risk management. This chapter discusses the nature of risk and uncertainty and how these considerations arise in the engineering of systems. The objectives of engineering risk management are described along with an overview of modern processes and practices. Last, new perspectives on managing risk in the engineering of systems-of-systems and enterprise systems are discussed.

Chapter 2 presents elements of probability theory — a topic foundational to understanding the nature of risk and ways to measure its chance of occurrence. Topics include the fundamental axioms and properties of probability. Basic concepts are emphasized along with how they apply to the analysis of risk in an engineering systems context.

Chapter 3 offers an introduction to decision analysis — a subject that owns much of its modern theoretical basis in the classic text *Decisions With Multiple Objectives: Preferences and Value Tradeoffs* by R. L. Keeney and H. Raiffa.

In Chapter 3, elements of preference theory and multiattribute value and utility function theory are explained. Concepts of value, utility, and risk functions are introduced, along with how they apply to measuring and managing risk in the engineering of systems.

Chapter 4 presents a series of essays on selected analytical topics that arise in engineering risk management. These topics were chosen due to the frequency with which they occur in practice and because they constitute the basics of any sound risk management process.

Topics in Chapter 4 include how to identify, write, and represent risks; methods to rank-order or prioritize risks in terms of their potential impacts to an engineering system project; and how to monitor progress in managing or mitigating a risk's potential adverse effects. In addition, two current and applied topics in engineering risk management are discussed.

The first topic is technical performance measures and how they can be used to monitor and track an engineering system's overall performance risk. For a system, these measures individually generate useful data; however, little has been developed in the engineering management community on how to integrate them into meaningful measures of performance risk — measures that can be readily tracked over time.

The second topic concludes the chapter with a discussion on risk management in the context of engineering enterprise systems. This is presented from a capability portfolio view. Applying, adapting, or defining risk management principles in an enterprise-wide problem space is truly the "cutting edge" of current practice.

The numerical precision shown in some of the book's examples, computations, and case discussions is intended only for illustrative and pedagogical purposes. In practice, analysts and engineers must always choose the level of precision appropriate to the nature of the problem being addressed. In systems engineering risk management, that seldom exceeds a single decimal point.

The book concludes with two appendixes. Appendix A presents a geometric approach for ranking risks. Appendix B presents success factors in systems engineering risk management. They come from my experience and those of my colleagues in industry, government, and academe.

Acknowledgments

I gratefully acknowledge a number of distinguished professors, engineers, and scientists who contributed to this book. Their insights have been instrumental in bringing about this work.

Academia

Tyson R. Browning, Ph.D., Assistant Professor of Enterprise Operations, Department of Information Systems and Supply Chain Management, Neeley School of Business, Texas Christian University, Fort Worth, Texas.

L. Robin Keller, Ph.D., Professor, Operations and Decision Technologies, The Paul Merage School of Business, University of California, Irvine, California; and Past-President, 2000–2002, Decision Analysis Society of the Institute for Operations Research and Management Sciences (INFORMS).

Craig W. Kirkwood, Ph.D., Professor, Chair, Department of Supply Chain Management, Arizona State University, Tempe, Arizona; and President, 2006–2008, Decision Analysis Society of INFORMS; and 2007 Ramsey medalist.

Jay Simon, Doctoral Student, Operations and Decision Technologies, The Paul Merage School of Business, University of California, Irvine, California.

K. Paul Yoon, Ph.D., Professor, Chairperson, Information Systems & Decision Sciences, Fairleigh Dickinson University, Teaneck, New Jersey. For correspondence with the author on the TOPSIS method, developed in 1981 with Ching-Lai Hwang. The TOPSIS method is summarized in Appendix A.

The MITRE Corporation

Bruce W. Lamar, Ph.D., created the *Max Average Rule* presented in Chapter 4.

Charlene J. McMahon and **Robert S. Henry,** whose experience and insights into success factors for engineering risk management are summarized in Appendix B.

Richard A. Moynihan, Ph.D., created the *Frequency Count Approach* presented in Chapter 4.

George Rebovich, Jr., authored the paper "Enterprise Systems Engineering Theory and Practice, Volume 2: Systems Thinking for the Enterprise: New and Emerging Perspectives," MITRE Paper MP 050000043, November 2005. Permission was granted to excerpt portions of this paper for Chapter 4.

Brian E. White, Ph.D., authored the paper "Fostering Intra-Organizational Communication of Enterprise Systems Engineering Practices," October 2006. Permission was granted to excerpt portions of this paper for Chapter 4.

Finally, I appreciate the staff at Chapman & Hall/CRC Press, Taylor & Francis Group, for their diligence, professionalism, and enthusiasm. I am grateful to David Grubbs, acquisition editor, and the production team for their efforts in bringing about this work.

Paul R. Garvey
Duxbury, Massachusetts

Chapter 1

Engineering Risk Management

1.1 Introduction

This chapter presents an introduction to engineering risk management. The nature of risk and uncertainty is discussed and how these considerations arise in the engineering of systems. The objectives of engineering risk management are described along with an overview of modern processes and practices. Last, new perspectives on managing risk in the engineering of systems-of-systems and enterprise systems are discussed.

Mentioned in the book's preface, engineering today's systems is sophisticated and complex. Increasingly, systems are being engineered by bringing together many separate systems that, as a whole, provide a capability otherwise not possible. Many systems no longer physically exist within clearly defined boundaries; rather, systems are more and more geographically and spatially distributed and interconnected through a rich and sophisticated set of networks and communications technologies.

These large-scale complex systems operate to satisfy a comparatively large set of users, stakeholders, or communities of interest. It is no longer enough to find just technology solutions to the engineering of these systems. Today's solutions must be adaptable to change, balanced with respect to expected performance, and risk managed while also considering the social, political, and economic environments within which the system will operate and evolve over time.

1.2 Engineering Risk Management Objectives

Engineering risk management is a program management process. At its best, engineering risk management is indistinguishable from program management.*

The objectives of engineering risk management are the early and continuous identification, management, and resolution of risks such that the engineering of a system is accomplished within cost, delivered on time, and meets user needs.

Why is engineering risk management important? There are many reasons. Below are five key considerations.

1. **Early and Continuous Risk Identification:** An engineering risk management program fosters the early and continuous identification of risks so options can be considered and actions implemented before risks seriously threaten a system's outcome objectives.

2. **Risk-Based Program Management:** Engineering risk management enables risk-informed decision-making and course-of-action planning throughout a program's development life cycle and particularly when options, alternatives, or opportunities need to be evaluated.

3. **Estimating and Justifying Risk Reserve Funds:** An engineering risk management program enables identified risk events to be mapped into a project's work breakdown structure. From this, the cost of their ripple effects can be estimated. Thus, an analytical justification can be established between a project's risk events and the amount of risk reserve (or contingency) funds that may be needed.

4. **Resource Allocation:** The analyses produced from an engineering risk management program will identify where management should consider allocating limited (or competing) resources to the most critical risks on an engineering system project.

5. **Situational Awareness and Risk Trends:** Engineering risk management can be designed to provide management with situational awareness in terms

* "Risk management is a method of managing that concentrates on identifying and controlling events that have a potential of causing unwanted change ... it is no more and no less than informed management" [Caver T. V., "Risk Management as a Means of Direction and Control," Fact Sheet, Program Managers Notebook, Defense Systems Management College (DSMC), Defense Acquisition University (DAU), No. 6.1, April 1985].

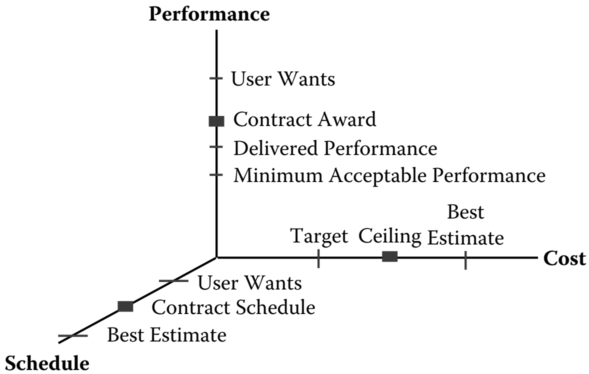


Figure 1.1: Pressures on a program manager's decision space.

of a project's risk status. This includes tracking the effectiveness of courses-of-action and trends in the rate that risks are closed with those newly identified and those that remain unresolved.

What are risks? Risks are events that, if they occur, will cause unwanted change in the cost, schedule, or technical performance of an engineering system. Thus, the occurrence of risk is an event that has negative consequences to an engineering system project. Risk is a probabilistic event; that is, risk is an event that may occur with probability p or may not occur with probability $(1 - p)$.

Why are there risks? Pressures to meet cost, schedule, and technical performance are the practical realities in engineering today's systems. Risk becomes present, in large-part, because expectations in these dimensions push what is technically or economically feasible. Managing risk is managing the inherent contention that exists within and across all these dimensions, as shown in Figure 1.1.

What is the goal of engineering risk management? Mentioned above, the goal is to identify cost, schedule, and technical performance risks early and continuously, such that control in any of these dimensions is not lost or the consequences on them are well understood. Risk management strives to enable risk-informed decision-making and investment planning throughout an engineering system's life cycle.

1.3 Overview of Process and Practice

This section presents an overview of engineering risk management process and practice, which varies greatly from very formal to very informal. The degree of practice is governed by management style, commitment, and a project team's "attitude" towards risk identification, analysis, and management. First, we'll begin with two definitions.

Definition 1.1 Risk is an event that, if it occurs, adversely affects the ability of a project to achieve its outcome objectives.

From this, a risk event has two aspects. The first is its occurrence probability. The second is its impact (or consequence) to an engineering system project. A general expression for this is given by Equation 1.1.

$$Risk = F(Probability, Impact) \tag{1.1}$$

In Chapter 4, we'll see how analyzing and prioritizing identified risk events must consider their occurrence probabilities and impacts (or consequences).

Definition 1.2 An event is uncertain if there is indefiniteness about its outcome.

Notice the distinction between the definition of risk and the definition of uncertainty. Risk is the chance of loss or injury. In a situation that includes favorable and unfavorable events, risk is the probability an unfavorable event occurs. Uncertainty is the indefiniteness about the outcome of a situation. We analyze uncertainty for the purpose of measuring risk. In an engineering system, the analysis might focus on measuring the risk of failing to achieve performance objectives, overrunning the budgeted cost, or delivering the system too late to meet user needs [1].

Why is the probability formalism used in risk management? Because a risk is a potential event, probability is used to express the chance the event will occur. Often, the nature of these events is such that subjective measures of probability are used in the analyses instead of objectively derived measures.

What is meant by a risk event's impact (or consequence)? How is it measured? Mentioned above, a risk event's consequence is typically expressed in terms of its impact on an engineering system's cost, schedule, and technical performance. However, there are often other important dimensions to consider. These include programmatic, social, political, and economic impacts.

Chapter 4 presents many ways consequence can be measured. Common methods include techniques from utility and value function theory (introduced in Chapter 3).

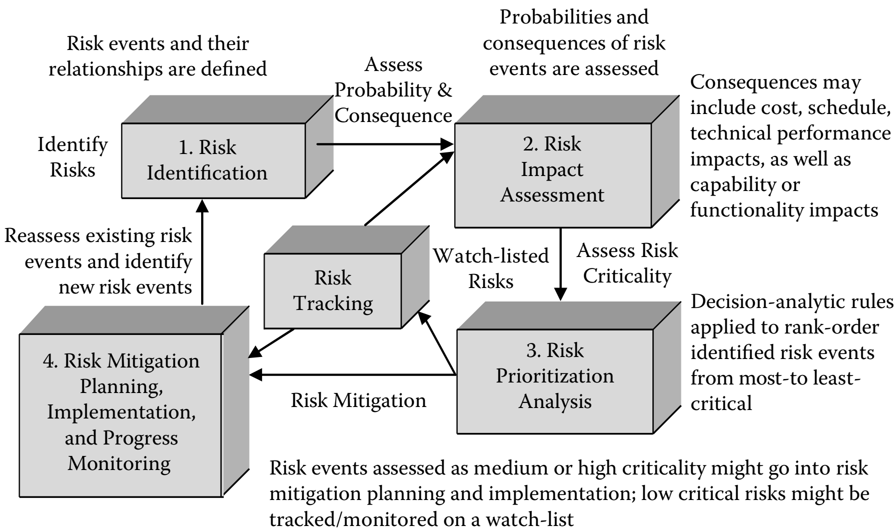


Figure 1.2: Steps common to a risk management process.

These formalisms enable risk events that impact a project in different types of units (e.g., dollars, months, processing speed) to be compared along normalized, dimensionless, scales. This is especially necessary when risk events are rank-ordered or prioritized on the basis of their occurrence probabilities and impacts.

In general, the risk management process can characterized according to steps shown in Figure 1.2.* [2] The following provides a brief description of each step.

Step 1. Risk Identification

Risk identification is the critical first step of the risk management process. Its objective is the *early and continuous* identification of risks to include those within and external** to the engineering system project. Mentioned above, these risks are events that, if they occur, have negative impacts on the project's ability to achieve performance or capability outcome goals.

* A similar view of the risk management process is presented in Chapter 4, Figure 4.1.

** Today, systems are increasingly being engineered to operate in networked environments. For systems such as these, it is important to identify risks from one system that may have negative collateral consequences to other systems within the network envelope. Here, the risk of one system failing to achieve its objectives may negatively impact the ability of other systems to achieve their objectives.

Step 2. Risk Impact or Consequence Assessment

In this step, an assessment is made of the impact each risk event could have on the engineering system project. Typically, this includes how the event could impact cost, schedule, or technical performance objectives. Impacts are not limited to only these criteria. Additional criteria such as political or economic consequences may also require consideration. An assessment is also made of the probability (chance) each risk event will occur. This often involves the use of subjective probability assessment techniques, particularly if circumstances preclude a direct evaluation of the probability by objective methods (i.e., engineering analysis, modeling, and simulation). Chapters 2 and 4 discuss the topic of subjective probability assessments.

Step 3. Risk Prioritization

At this step, the overall set of identified risk events, their impact assessments, and their occurrence probabilities are "processed" to derive a most-to least-critical rank-order of identified risks. Decision analytic techniques such as utility theory, value function theory, or ordinal ranking techniques are formalisms often used to derive a most-to least-critical rank-order of identified risks.

A major purpose for prioritizing risks is to form a basis for allocating critical resources. These resources include the assignment of additional personnel and funding (if necessary) to focus on resolving risks deemed most critical to the engineering system project.

Step 4. Risk Mitigation Planning

This step involves the development of mitigation plans designed to manage, eliminate, or reduce risk to an acceptable level. Once a plan is implemented, it is continually monitored to assess its efficacy with the intent of revising the courses-of-action if needed. Chapter 4 discusses ways to monitor mitigation plan progress.

1.4 New Perspectives on Engineering Systems

Today, the body of literature on engineering risk management is largely aimed at addressing traditional engineering system projects — those systems designed and engineered against a set of well-defined user requirements, specifications, and technical standards. In contrast, little exists in the community on how risk management principles apply to a system whose entire functionality is governed by the interaction of a set of highly interconnected supporting systems. Such systems may be referred to as systems-of-systems.

A system-of-systems can be thought of as a set or arrangement of systems that are related or interconnected to provide a given *capability* that otherwise would not be possible. The loss of any part of supporting systems degrades the performance or capabilities of the whole [3].

An example of a system-of-systems could be interdependent information systems. While individual systems within the system-of-systems may be developed to satisfy the needs of a given user or group of users, the information they share is so important that the loss of a single system may deprive other systems of the data needed to achieve even minimal capabilities [3].

Shown in Figure 1.3, a system-of-systems can be decomposed into its individual systems. These individual systems can then be decomposed into their individual subsystems. The result of this process produces a tree-like hierarchical structure.

What makes risk management in the engineering of systems-of-systems more challenging than managing risk in a traditional engineering system project? How does the delivery of capability over time effect how risks are managed in a system-of-systems?

With regards to the first question, the difference is principally a matter of scope. From a high-level perspective, the basic risk management process steps are

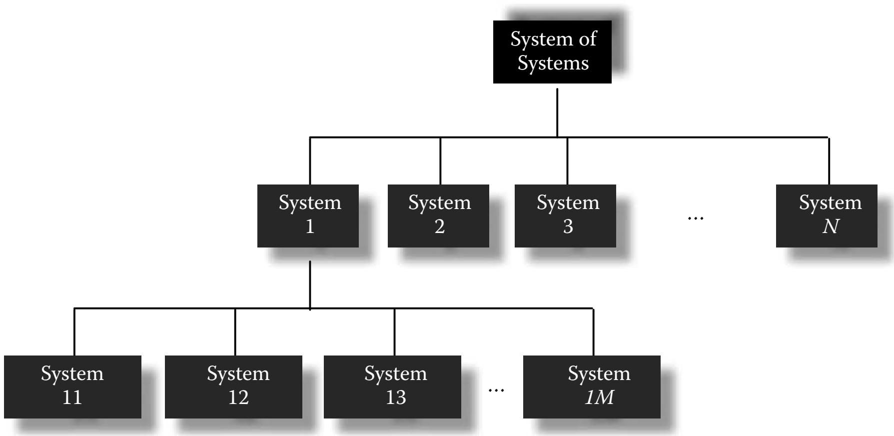


Figure 1.3: An illustrative system-of-systems hierarchy.

the same. The challenge comes from implementing and managing these steps across a large-scale complex system-of-systems — one whose subordinate systems, managers, and stakeholders may be geographically dispersed, organizationally distributed, and may not have fully intersecting user needs.

In answer to the second question, the difference is in aligning or mapping identified risks to capabilities planned to be delivered within a specified build by a specified time. Here, it is critically important that risk impact assessments are made as a function of what capabilities are affected, when these effects occur, and their impacts on users and stakeholders.

Lack of clearly defined system boundaries, management lines of responsibility, and accountability further challenges the management of risk in the engineering of systems-of-systems. These challenges are even greater when a system-of-systems is delivering capabilities along an evolutionary build strategy. Team-building and user and stakeholder acceptance of risk management, and their participation in the process, are among the organizational and social considerations essential for success.

Given the above, management needs to establish a trustworthy environment where the reporting of risks and their potential consequences is encouraged and rewarded. Without this, management will have an incomplete picture of risks. Risks that truly threaten the successful engineering of a system-of-systems may become evident only when it is too late to effectively manage or mitigate them.

Mentioned earlier, a system-of-systems is often planned and engineered to deliver capabilities through a series of evolutionary builds. Thus, identified risks can originate from many different source points and threaten the system-of-systems at different points in time. Furthermore, these risks (and their source points) must be mapped to the capabilities they potentially affect, according to their planned delivery year. Assessments must then be made of each risk's potential impacts to planned capabilities and whether these consequences have unwanted collateral effects on other dependent capabilities or technologies.

Figure 1.4 illustrates types of displays a system-of-systems risk management process might generate. In the upper-left, the risk situation is displayed as a function of each risk event's occurrence probability and its overall impact or consequence score (Chapter 4 presents ways to compute these scores).

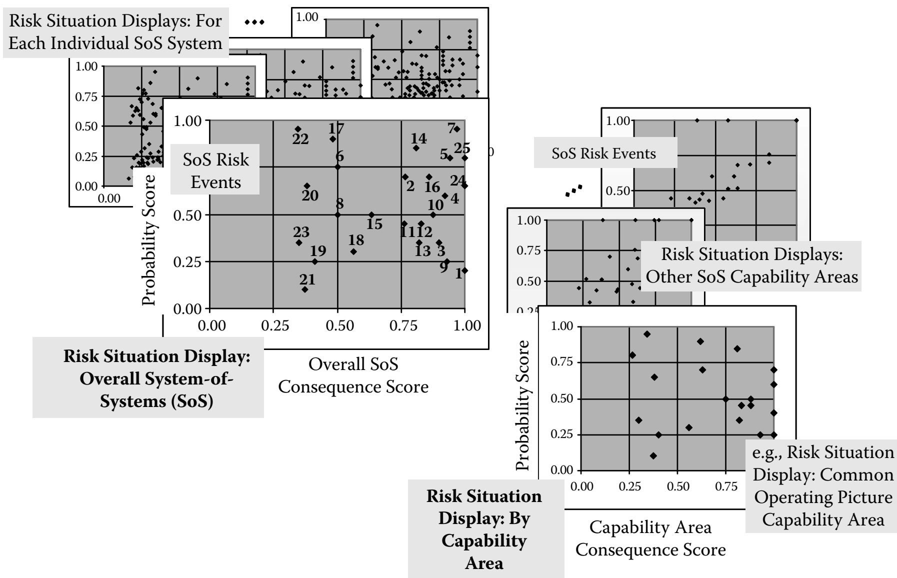


Figure 1.4: Example risk situation visuals and displays.

From a system-of-systems perspective, the overall risk situation is not a simple "rollup" of its subordinate system-level risks. Rather, it is a fusing of certain lower-level individual system risks that have the potential to adversely impact the system-of-systems if these risks occur. So, some risks may remain managed at the individual system levels while others are potentially serious enough to warrant the attention of system-of-systems leadership and, thus, be elevated accordingly.

The lower-right element in Figure 1.4 illustrates one way to correlate risks to capabilities being threatened. Suppose Figure 1.4 shows risk events that, if they occur, threaten the capability area "Common Operating Picture." Similar displays could be generated for each system-of-systems capability (or capability area) planned for delivery by specified builds and by specified dates.

This chapter ends with a brief commentary on an even greater challenge than that posed by the preceding discussion. That challenge is the management of risk in the context of engineering enterprise systems.

Enterprise systems engineering is truly an emerging discipline. It encompasses and extends "traditional" systems engineering to create and evolve "webs" of systems and systems-of-systems that operate in a network-centric way to deliver capabilities via services, data, and applications through a richly interconnected network of information and communications technologies. This is systems engineering at today's "cutting edge." Enterprise environments (such as the Internet) offer users ubiquitous cross-boundary access to wide varieties of services, applications, and information repositories.

In an enterprise context, risk management is envisioned as an integration of people, processes, and tools that together ensure the early and continuous identification and resolution of enterprise risks. The goal is to provide decision-makers with an enterprise-wide understanding of risks, their potential consequences, interdependencies, and rippling effects within and beyond enterprise "boundaries." Ultimately, risk management aims to establish and maintain a holistic view of risks across the enterprise, so capabilities and performance objectives are achieved via risk-informed resource and investment decisions.

Today, we're in the early stage of understanding how systems engineering, engineering management, and social science methods weave together to create systems that "live" and "evolve" in enterprise environments. Chapter 4 discusses some of these understandings, specifically as they pertain to risk management. The analytical practices discussed will themselves evolve as the community gains experience and knowledge about engineering in the enterprise problem space.

Questions and Exercises

1. In Section 1.2, engineering risk management was described as a program management process and one that, at its best, is *indistinguishable* from program management.
   (a) Discuss how one might institute protocols to ensure risk management and program management are inseparable disciplines in the design and engineering of systems.
   (b) What leadership qualities are needed in the management environment to accomplish (a) above?

2. In Section 1.2, the aim of engineering risk management was described as the *early and continuous* identification, management, and resolution of risks such that the engineering of a system is accomplished within cost, delivered on time, and meets user needs. Discuss protocols needed on an

engineering system project to ensure early and continuous identification of risks throughout a project's life cycle.

3. From the discussion in Section 1.2, if A is a *risk event* then why must the probability of A's occurring be strictly greater than zero *and* also strictly less than one?

4. Section 1.2 discussed how pressures to meet cost, schedule, and technical performance are the practical realities in engineering today's systems. Risk becomes present, in large part, because expectations in these dimensions push what is technically or economically feasible. Discuss ways an engineering manager might lessen or guard against these pressures within and across these dimensions. For convenience, Figure 1.1 is repeated below.

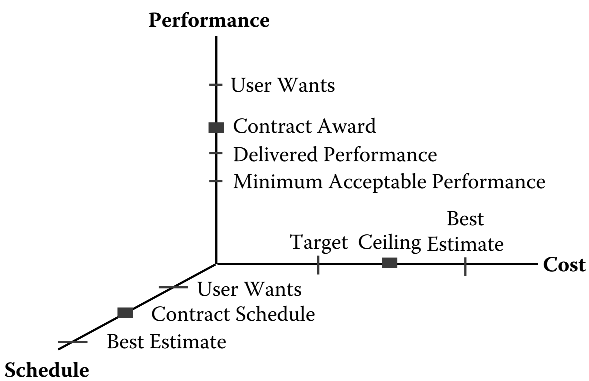


Figure 1.1: Pressures on a program manager's decision space.

5. Section 1.3 presented a risk management process with steps considered generally the same for most engineering system projects. This process was illustrated in Figure 1.2, which is repeated below for convenience.
   (a) Discuss how this process might be tailored for use in a risk management program designed for engineering a system-of-systems.
   (b) Discuss how this process might be tailored for use in a risk management program designed for engineering an *enterprise system*, one that consists of a web of systems and systems-of-systems.

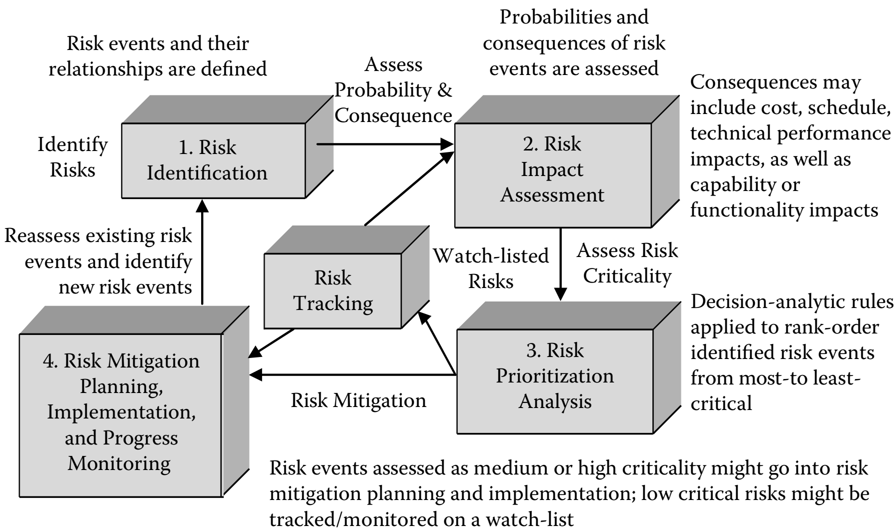


Figure 1.2: Steps common to a risk management process.

References

[1] Garvey, P. R., 2000. *Probability Methods for Cost Uncertainty Analysis: A Systems Engineering Perspective*, Marcel Dekker, New York.

[2] Garvey, P. R., January 2005. "System-of-Systems Risk Management: Perspectives on Emerging Process and Practice," The MITRE Corporation, MP 04B0000054.

[3] Chairman of the Joint Chiefs of Staff Manual (CJCSM 3170.01), 24 June 2003.

Chapter 2

Elements of Probability Theory

2.1 Introduction

Whether referring to a storm's intensity, an arrival time, or the success of a decision, the word "probable," or "likely," has long been part of our language. Most people have an appreciation for the impact of chance on the occurrence of an event. In the last 350 years, the theory of probability has evolved to explain the nature of chance and how it can be studied.

Probability theory is the formal study of events whose outcomes are uncertain. Its origins trace to 17th-century gambling problems. Games that involved playing cards, roulette wheels, and dice provided mathematicians with a host of interesting problems. The solutions to many of these problems yielded the first principles of modern probability theory. Today, probability theory is of fundamental importance in science, engineering, and business.

Engineering risk management aims to identify and manage events whose outcomes are uncertain. In particular, its focus is on events that, if they occur, have unwanted impacts or consequences to a project or program. The phrase "*if they occur*" means these events are probabilistic in nature. Thus, understanding them in the context of probability concepts is essential. This chapter presents an introduction to these concepts and illustrates how they apply to managing risks in engineering systems.

2.2 Interpretations and Axioms

We begin this discussion with the traditional look at dice. If a six-sided die is tossed, there clearly are six possible outcomes for the number that appears on the upturned face. These outcomes can be listed as elements in a set $\{1, 2, 3, 4, 5, 6\}$.

The set of all possible outcomes of an experiment, such as tossing a six-sided die, is called the *sample space*, which we will denote by Ω. The individual outcomes of Ω are called sample points, which we will denote by ω.

An *event* is any subset of the sample space. An event is *simple* if it consists of exactly one outcome. Simple events are also referred to as *elementary* events or elementary outcomes. An event is *compound* if it consists of more than one outcome. For instance, let A be the event an odd number appears and B be the event an even number appears in a single toss of a die. These are compound events that can be expressed by the sets $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$. Event A occurs *if and only if* one of the outcomes in A occurs. The same is true for event B.

Seen in this discussion, events can be represented by sets. New events can be constructed from given events according to the rules of set theory. The following presents a brief review of set theory concepts.

Union. For any two events A and B of a sample space, the new event $A \cup B$ (which reads A union B) consists of all outcomes either in A or in B or in both A and B. The event $A \cup B$ occurs if either A or B occurs. To illustrate the union of two events, consider the following: if A is the event an odd number appears in the toss of a die and B is the event an even number appears, then the event $A \cup B$ is the set $\{1, 2, 3, 4, 5, 6\}$, which is the sample space for this experiment.

Intersection. For any two events A and B of a sample space Ω, the new event $A \cap B$ (which reads A *intersection* B) consists of all outcomes that are in *both* A and in B. The event $A \cap B$ occurs *only if both* A *and* B *occur*. To illustrate the intersection of two events, consider the following: if A is the event a 6 appears in the toss of a die, B is the event an odd number appears, and C is the event an even number appears, then the event $A \cap C$ is the simple event $\{6\}$; on the other hand, the event $A \cap B$ contains no outcomes. Such an event is called the *null event*. The null event is traditionally denoted by $\emptyset$. In general, if $A \cap B = \emptyset$, we say events A and B are *mutually exclusive* (*disjoint*). For notation convenience, the intersection of two events A and B is sometimes written as AB, instead of $A \cap B$.

Complement. The complement of event A, denoted by A^c, consists of all outcomes in the sample space Ω that are not in A. The event A^c occurs *if and only if* A does not occur. The following illustrates the complement of an event. If C is the event an even number appears in the toss of a die, then C^c is the event an odd number appears.

Subset. Event A is said to be a subset of event B if all the outcomes in A are also contained in B. This is written as $A \subset B$.

In the preceding discussion, the sample space for the toss of a die was given by $\Omega = \{1, 2, 3, 4, 5, 6\}$. If we *assume* the die is fair, then any outcome in the sample space is as likely to appear as any other. Given this, it is reasonable to conclude the proportion of time each outcome is expected to occur is 1/6. Thus, the probability of each simple event in the sample space is

$$P(\{1\}) = P(\{2\}) = P(\{3\}) = P(\{4\}) = P(\{5\}) = P(\{6\}) = \frac{1}{6}$$

Similarly, suppose B is the event an odd number appears in a single toss of the die. This compound event is given by the set $B = \{1, 3, 5\}$. Since there are three ways event B can occur out of six possible, the probability of event B is

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

The following presents a view of probability known as the equally likely interpretation.

Equally Likely Interpretation. In this view, if a sample space Ω consists of a finite number of outcomes n, which are all equally likely to occur, then the probability of each simple event is $1/n$. If an event A consists of m of these n outcomes, then the probability of event A is

$$P(A) = \frac{m}{n} \tag{2.1}$$

In the above, it is assumed the sample space consists of a *finite* number of outcomes and all outcomes are equally likely to occur. What if the sample space is finite but the outcomes are *not* equally likely? In these situations, probability might be measured in terms of how frequently a particular outcome occurs when the experiment is repeatedly performed under identical conditions. This leads to a view of probability known as the frequency interpretation.

Frequency Interpretation. In this view, the probability of an event is the limiting proportion of time the event occurs in a set of n repetitions of the experiment. In particular, we write this as

$$P(A) = \lim_{n \to \infty} \frac{n(A)}{n}$$

where $n(A)$ is the number of times in n repetitions of the experiment the event A occurs. In this sense $P(A)$ is the limiting frequency of event A. Probabilities measured by the frequency interpretation are referred to as *objective probabilities*. In many circumstances it is appropriate to work with objective probabilities. However, there are limitations with this interpretation of probability. It restricts events to those that can be subjected to repeated trials conducted under *identical conditions*. Furthermore, it is not clear how many trials of an experiment are needed to obtain an event's limiting frequency.

Axiomatic Definition. In 1933, the Russian mathematician A.N. Kolmogorov* presented a definition of probability in terms of three axioms [1]. These axioms define probability in a way that encompasses the *equally likely and frequency interpretations* of probability. It is known as the axiomatic definition of probability. It is the view of probability adopted in this book. Under this definition, it is assumed for each event A, in the sample space Ω, there is a real number $P(A)$ that denotes the probability of A. In accordance with Kolmogorov's axioms, a probability is simply a numerical measure that satisfies the following:

Axiom 1 $0 \leq P(A) \leq 1$ for any event A in Ω

Axiom 2 $P(\Omega) = 1$

Axiom 3 For any sequence of mutually exclusive events $A_1, A_2, \ldots$ defined on Ω it follows that $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$
For any *finite sequence* of mutually exclusive events $A_1, A_2, \ldots, A_n$ defined on Ω it follows that $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

The first axiom states the probability of any event is a non-negative number in the interval zero to one. In axiom 2, the sample space Ω is sometimes referred to as the *sure* or *certain event*; therefore, we have $P(\Omega)$ equal to one. Axiom 3 states for any sequence of mutually exclusive events, the probability of at least one event occurring is the sum of the probabilities associated with each event A_i. In axiom 3, this sequence may also be finite. From these axioms come five basic theorems of probability.

*A. N. Kolmogorov, *Grundbegriffe der Wahrscheinlichkeitsrechnung*, *Ergeb. Mat. und ihrer Grenzg.*, vol. 2, no. 3, 1933. Translated into English by N. Morrison, *Foundations of the Theory of Probability*, New York (Chelsea), 1956 [1].

Theorem 2.1 *The probability event A occurs is one minus the probability it will not occur; that is,*

$$P(A) = 1 - P(A^c)$$

Theorem 2.2 *The probability associated with the null event ∅ is zero; that is,*

$$P(\emptyset) = 0$$

Theorem 2.3 *If events A_1 and A_2 are mutually exclusive, then*

$$P(A_1 \cap A_2) \equiv P(A_1 A_2) = 0$$

Theorem 2.4 *For any two events A_1 and A_2*

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

Theorem 2.5 *If event A_1 is a subset of event A_2 then*

$$P(A_1) \leq P(A_2)$$

Measure of Belief Interpretation. From the axiomatic view, probability need only be a numerical measure satisfying the three axioms stated by Kolmogorov. Given this, it is possible for probability to reflect a "measure of belief" in an event's occurrence. For instance, an engineer might assign a probability of 0.70 to the event "the radar software for the Advanced Air Traffic Control System (AATCS) will not exceed 100K delivered source instructions." We consider this event to be non-repeatable. It is not practical, or possible, to build the AATCS n-times (and under identical conditions) to determine whether this probability is indeed 0.70. When an event such as this arises, its probability may be assigned. Probabilities assigned on the basis of personal judgment, or measure of belief, are known as *subjective probabilities*.

Subjective probabilities are the most common in engineering system projects. Such probabilities are typically assigned by expert technical judgment. The engineer's probability assessment of 0.70 is a subjective probability. Ideally, subjective probabilities should be based on available evidence and previous experience with similar events. Subjective probabilities become suspect if they are premised on limited insights or no prior experience. Care is also needed in soliciting subjective probabilities. They must certainly be plausible *and* they must be *consistent* with Kolmogorov's axioms and the theorems of probability, which stem from these axioms. Consider the following:

The XYZ Corporation has offers on two contracts A and B. Suppose the proposal team made the following subjective probability assignments. The chance of winning contract A is 40%, the chance of winning contract B is 20%, the chance of winning *contract A or contract B* is 60%, and the chance of winning *both contract A and contract B* is 10%. It turns out this set of probability assignments is *not* consistent with the axioms and theorems of probability. Why is this?* If the chance of winning contract B was changed to 30%, then this *set of probability assignments* would be consistent.

Kolmogorov's axioms, and the resulting theorems of probability, do not suggest how to assign probabilities to events. Instead, they provide a way to verify that probability assignments are consistent, whether these probabilities are objective or subjective.

Risk versus Uncertainty. There is an important distinction between the terms *risk* and *uncertainty*. Risk is the chance of loss or injury. In a situation that includes favorable and unfavorable events, risk is the *probability an unfavorable event occurs*. Uncertainty is the *indefiniteness about the outcome of a situation*. We analyze uncertainty for the purpose of measuring risk. In systems engineering the analysis might focus on measuring the risk of: (1) failing to achieve performance objectives, (2) overrunning the budgeted cost, or (3) delivering the system too late to meet user needs. Conducting the analysis often involves degrees of subjectivity. This includes defining the events of concern and, when necessary, subjectively specifying their occurrence probabilities. Given this, it is fair to ask whether it is meaningful to apply rigorous mathematical procedures to such analyses. In a speech before the 1955 Operations Research Society of America meeting, Charles J. Hitch (RAND) addressed this question. He stated [2, 3]:

> Systems analyses provide a framework which permits the judgment of experts in many fields to be combined to yield results that transcend any individual judgment. The systems analyst may have to be content with better rather than optimal solutions; or with devising and costing sensible methods of hedging; or merely with discovering critical sensitivities. We tend to be worse, in an absolute sense, in applying analysis or scientific method to broad context problems; but unaided intuition in such problems is also much worse in the absolute sense. Let's not deprive ourselves of any useful tools, however short of perfection they may fail.

*The answer can be seen from theorem 2.4.

2.3 Conditional Probability and Bayes' Rule

In many circumstances, the probability of an event is conditioned on knowing another event has taken place. Such a probability is known as a *conditional probability*. *Conditional probabilities* incorporate information about the occurrence of another event. The conditional probability of event A given event B has occurred is denoted by $P(A\,|B)$. If a pair of dice is tossed, then the probability the sum of the toss is *even* is 1/2. This probability is known as a *marginal* or *unconditional probability*.

How would this unconditional probability change (i.e., be conditioned) if it was *known* the sum of the toss was a number less than 10? This is discussed in the following example.

Example 2.1
A pair of dice is tossed and the sum of the toss is a number less than 10. Given this, compute the probability this sum is an even number.

Solution
Suppose we define events A and B as follows:

A: The sum of the toss is even

B: The sum of the toss is a number less than 10

The sample space Ω contains 36 possible outcomes; however, in this case we want the subset of Ω containing *only* those outcomes whose toss yielded a sum less than 10. This subset is shown in Table 2.1. It contains 30 outcomes. Within Table 2.1, only 14 outcomes are associated with the event "the sum of the toss is even given the sum of the toss is a number less than 10."

$$\left\{\begin{matrix}(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5)\\ (4, 2), (4, 4), (5, 1), (5, 3), (6, 2)\end{matrix}\right\}$$

Therefore, the probability of this event is $P(A\,|B) = 14/30$

In Example 2.1, observe that $P(A\,|B)$ was obtained directly from a subset of the sample space Ω and that $P(A\,|B) = 14/30 < P(A) = 1/2$.

If A and B are events in the same sample space Ω, then $P(A\,|B)$ is the probability of event A within the subset of the sample space defined by event B.

TABLE 2.1: Outcomes Associated with Event B

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	**(2,2)**	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	**(3,3)**	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	**(4,4)**	(4,5)	
(5,1)	(5,2)	(5,3)	(5,4)		
(6,1)	(6,2)	(6,3)			

A subset of Ω that contains only those outcomes whose toss yielded a sum less than 10

Formally, the *conditional probability of event A given event B has occurred*, where $P(B) > 0$, is defined as

$$P(A\,|B) = \frac{P(A \cap B)}{P(B)} \tag{2.2}$$

Likewise, the conditional probability of event B given event A has occurred, where $P(A) > 0$, is defined as

$$P(B\,|A) = \frac{P(B \cap A)}{P(A)} \tag{2.3}$$

Example 2.2

A proposal team from XYZ Corporation has offers on two contracts A and B. The team made subjective probability assignments on the chances of winning these contracts. They assessed a 40% chance on the event winning contract A, a 50% chance on the event winning contract B, and a 30% chance on the event winning both contracts. Given this, what is the probability of:

(a) Winning at least one contract?

(b) Winning contract A and not winning contract B?

(c) Winning contract A if the proposal team has won at least one contract?

Solution

(a) Winning at least one contract means winning either contract A *or* contract B or both contracts. This event is represented by the set $A \cup B$. From theorem 2.4

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

therefore

$$P(A \cup B) = 0.40 + 0.50 - 0.30 = 0.60$$

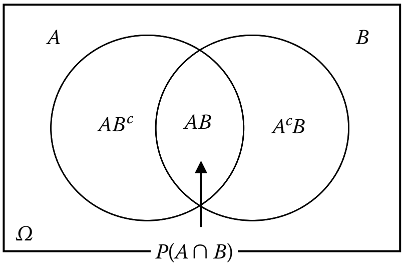


Figure 2.1: Venn diagram for $P(A) = P((A \cap B^c) \cup (A \cap B))$.

(b) The event winning contract A and not winning contract B is represented by the set $A \cap B^c$. From the Venn diagram in Figure 2.1, observe that

$$P(A) = P((A \cap B^c) \cup (A \cap B))$$

Since the events $A \cap B^c$ and $A \cap B$ are mutually exclusive (disjoint), from theorem 2.3 and theorem 2.4 we have

$$P(A) = P(A \cap B^c) + P(A \cap B)$$

This is equivalent to

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

therefore,

$$P(A \cap B^c) = P(A) - P(A \cap B) = 0.40 - 0.30 = 0.10$$

(c) If the proposal team has won one of the contracts, the probability of winning contract A must be revised (or conditioned) on this information. This means we must compute $P(A \mid A \cup B)$. From Equation 2.2

$$P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

Since

$$P(A) = P(A \cap (A \cup B))$$

we have

$$P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{0.40}{0.60} = \frac{2}{3} \approx 0.67$$

A consequence of conditional probability is obtained if we multiply Equations 2.2 and 2.3 by $P(B)$ and $P(A)$, respectively. This multiplication yields

$$P(A \cap B) = P(B)P(A\,|B) = P(A)P(B\,|A) \tag{2.4}$$

Equation 2.4 is known as the *multiplication rule*. The multiplication rule provides a way to express the probability of the intersection of two events in terms of their conditional probabilities. An illustration of this rule is presented in example 2.3.

Example 2.3

A box contains memory chips of which 3 are defective and 97 are non-defective. Two chips are drawn at random, one after the other, without replacement. Determine the probability:

(a) Both chips drawn are defective.

(b) The first chip is defective and the second chip is non-defective.

Solution

(a) Let A and B denote the event the first and second chips drawn from the box are *defective*, respectively. From the multiplication rule, we have

$$\begin{aligned} P(A \cap B) &= P(A)P(B\,|A) \\ &= P(\text{1st chip defective})\, P(\text{2nd chip defective}|\text{1st chip defective}) \\ &= \frac{3}{100}\left(\frac{2}{99}\right) = \frac{6}{9900} \end{aligned}$$

(b) To determine the probability the first chip drawn is defective and the second chip is *non-defective*, let C denote the event the second chip drawn is non-defective. Thus,

$$\begin{aligned} P(A \cap C) &= P(AC) = P(A)P(C\,|A) \\ &= P(\text{1st chip defective})\, P(\text{2nd chip nondefective}|\text{1st chip defective}) \\ &= \frac{3}{100}\left(\frac{97}{99}\right) = \frac{291}{9900} \end{aligned}$$

In this example the sampling was performed *without replacement*. Suppose the chips sampled were *replaced*; that is, the first chip selected was replaced before the second chip was selected. In that case, the probability of a defective chip being selected on the second drawing is independent of the outcome of the first chip drawn. Specifically,

$$P(\text{2nd chip defective}) = P(\text{1st chip defective}) = 3/100$$

so

$$P(A \cap B) = \frac{3}{100}\left(\frac{3}{100}\right) = \frac{9}{10000}$$

and

$$P(A \cap C) = \frac{3}{100}\left(\frac{97}{100}\right) = \frac{291}{10000}$$

Independent Events

Two events A and B are said to be *independent* if and only if

$$P(A \cap B) = P(A)P(B) \tag{2.5}$$

and *dependent* otherwise. Events $A_1, A_2, \ldots, A_n$ are (mutually) *independent* if and only if for every set of indices $i_1, i_2, \ldots, i_k$ between 1 and n, inclusive,

$$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2})\ldots P(A_{i_k}), \quad (k = 2, \ldots, n)$$

For instance, events A_1, A_2, and A_3, are independent (or mutually independent) if the following equations are satisfied

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) \tag{2.5a}$$

$$P(A_1 \cap A_2) = P(A_1)P(A_2) \tag{2.5b}$$

$$P(A_1 \cap A_3) = P(A_1)P(A_3) \tag{2.5c}$$

$$P(A_2 \cap A_3) = P(A_2)P(A_3) \tag{2.5d}$$

It is possible to have three events A_1, A_2, and A_3 for which Equations 2.5b through 2.5d hold but Equation 2.5a does not hold. Mutual independence implies pairwise independence, in the sense that Equations 2.5b through 2.5d hold, but the converse is not true.

There is a close relationship between independent events and conditional probability. To see this, suppose events A and B are independent. This implies

$$P(AB) = P(A)P(B)$$

From this, Equations 2.2 and 2.3 become, respectively, $P(A\,|B) = P(A)$ and $P(B\,|A) = P(B)$. Thus, when two events are independent the occurrence of one event has no impact on the probability the other event occurs.

To illustrate the concept of independence, suppose a fair die is tossed. Let A be the event an odd number appears. Let B be the event one of these numbers {2, 3, 5, 6} appears. From this,

$$P(A) = 1/2$$

and

$$P(B) = 2/3$$

Since $A \cap B$ is the event represented by the set $\{3, 5\}$, we can readily state $P(A \cap B) = 1/3$. Therefore, $P(A \cap B) = P(AB) = P(A)P(B)$ and we conclude events A and B are independent.

Dependence can be illustratedby tossing two fair dice. Suppose A is the event the sum of the toss is odd and B is the event the sum of the toss is even. Here, $P(A \cap B) = 0$ and $P(A)$ and $P(B)$ were each 1/2. Since $P(A \cap B) \neq P(A)P(B)$ we would conclude events A and B are dependent, in this case.

It is important not to confuse the meaning of independent events with mutually exclusive events as shown in Figure 2.2. If events A and B are mutually exclusive, the event *A and B* is empty; that is, $A \cap B = \emptyset$. This implies $P(A \cap B) = P(\emptyset) = 0$. If events A and B are *independent* with $P(A) \neq 0$ and $P(B) \neq 0$, then A and B cannot be mutually exclusive since $P(A \cap B) = P(A)P(B) \neq 0$.

Bayes' Rule

Suppose we have a collection of events A_i representing possible conjectures about a topic. Furthermore, suppose we have some initial probabilities associated with the "truth" of these conjectures. Bayes' rule* provides a way to update (or revise) initial probabilities when new information about these conjectures is evidenced.

*Named in honor of Thomas Bayes (1702–1761), an English minister and mathematician.

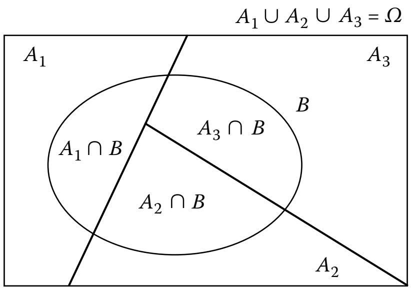


Figure 2.2: Partitioning Ω into three mutually exclusive sets.

Bayes' rule is a consequence of conditional probability. Suppose we partition a sample space Ω into a finite collection of three mutually exclusive events as shown in Figure 2.2. Define these events as A_1, A_2, and A_3 where $A_1 \cup A_2 \cup A_3 = \Omega$. Let B denote an arbitrary event contained in Ω. We can write the event B as

$$B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B)$$

Since the events $(A_1 \cap B)$, $(A_2 \cap B)$, $(A_3 \cap B)$ are mutually exclusive, we can apply axiom 3 and write

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$$

From the multiplication rule given in Equation 2.4, $P(B)$ can be expressed in terms of conditional probability as

$$P(B) = P(A_1)P(B\,|A_1) + P(A_2)P(B\,|A_2) + P(A_3)P(B\,|A_3)$$

This equation is known as the *total probability law*. Its generalization is

$$P(B) = \sum_{i=1}^{n} P(A_i)P(B\,|A_i)$$

where $\Omega = \bigcup\limits_{i=1}^{n} A_i$ and $A_i \cap A_j = \emptyset$ and $i \neq j$.

The conditional probability for each event A_i given event B has occurred is

$$P(A_i\,|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B\,|A_i)}{P(B)}$$

When the total probability law is applied to this equation we have

$$P(A_i\,|B) = \frac{P(A_i)P(B\,|A_i)}{\sum_{i=1}^{n} P(A_i)P(B\,|A_i\,)} \tag{2.6}$$

Equation 2.6 is known as *Bayes' Rule*.

Example 2.4

The ChipyTech Corporation has three divisions D_1, D_2, and D_3 that each manufacture a specific type of microprocessor chip. From the total annual output of chips produced by the corporation, D_1 manufactures 35%, D_2 manufactures 20%, and D_3 manufactures 45%. Data collected from the quality control group indicate 1% of the chips from D_1 are defective, 2% of the chips from D_2 are defective, and 3% of the chips from D_3 are defective. Suppose a chip was randomly selected from the total annual output produced and it was found to be defective. What is the probability it was manufactured by D_1? By D_2? By D_3?

Solution

Let A_i denote the event the selected chip was produced by division $D_i (i = 1, 2, 3)$. Let B denote the event the selected chip is defective. To determine the probability the defective chip was manufactured by D_i we must compute the conditional probability $P(A_i\,|B)$ for $i = 1, 2, 3$. From the information provided, we have

$$P(A_1) = 0.35, \quad P(A_2) = 0.20, \quad \text{and} \quad P(A_3) = 0.45$$
$$P(B|A_i) = 0.01, \quad P(B|A_2) = 0.02, \quad P(B|A_3) = 0.03$$

The total probability law and Bayes' rule will be used to determine $P(A_i\,|B)$ for each i = 1, 2, and 3. Recall that $P(B)$ can be written as

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$
$$P(B) = 0.35(0.01) + 0.20(0.02) + 0.45(0.03) = 0.021$$

and from Bayes' rule we can write

$$P(A_i\mid B) = \frac{P(A_i)P(B\,|A_i)}{\sum_{i=1}^{n} P(A_i)P(B\,|A_i\,)} = \frac{P(A_i)P(B\,|A_i)}{P(B)}$$

TABLE 2.2: Bayes' Probability Updating: Example 2.4 Summary

i	$P(A_i)$	$P(A_i \mid B)$
1	0.35	0.167
2	0.20	0.190
3	0.45	0.643

from which

$$P(A_1 \mid B) = \frac{P(A_1)P(B \mid A_1)}{P(B)} = \frac{0.35(0.01)}{0.021} = 0.167$$

$$P(A_2 \mid B) = \frac{P(A_2)P(B \mid A_2)}{P(B)} = \frac{0.20(0.02)}{0.021} = 0.190$$

$$P(A_3 \mid B) = \frac{P(A_3)P(B \mid A_3)}{P(B)} = \frac{0.45(0.03)}{0.021} = 0.643$$

Table 2.2 provides a comparison of $P(A_i)$ with $P(A_i \mid B)$ for each $i = 1, 2, 3$.

The probabilities given by $P(A_i)$ are the probabilities the selected chip will have been produced by division D_i before it is randomly selected and before it is known whether the chip is defective. Therefore, $P(A_i)$ are the *prior*, or *a priori* (before-the-fact) probabilities. The probabilities given by $P(A_i \mid B)$ are the probabilities the selected chip was produced by division D_i after it is known the selected chip is defective. Therefore, $P(A_i \mid B)$ are the *posterior*, or *a posteriori* (after-the-fact) probabilities. Bayes' rule provides a means for the computation of posterior probabilities from the known prior probabilities $P(A_i)$ and the conditional probabilities $P(B \mid A_i)$ for a particular situation or experiment.

Bayes' rule established areas of study that became known as *Bayesian inference* and *Bayesian decision theory*. These areas play important roles in the application of probability theory to systems engineering problems. In the total probability law, we can think of A_i as representing possible states of nature to which an engineer assigns subjective probabilities. These subjective probabilities are the prior probabilities, which are often premised on personal judgments based on past experience. In general, Bayesian methods offer a powerful way to revise or update probability assessments as new information becomes available.

2.4 Applications to Engineering Risk Management

Chapter 2 concludes with an expanded discussion of Bayes' rule in terms of its application to the analysis of risks in the engineering of systems. In addition, a best-practice protocol for expressing risk in terms of its occurrence probability and consequences is introduced.

2.4.1 Probability Inference — An Application of Bayes' Rule

This discussion presents a technique known as *Bayesian inference*. Bayesian inference is a way to examine how an initial belief in the truth of a hypothesis H may change when evidence e relating to it is observed. This is done by an application of Bayes' rule, which we illustrate in the discussion below.

Suppose an engineering firm has been awarded a project to develop a software application. Suppose a number of challenges are associated with this, among them (1) staffing the project, (2) managing multiple development sites, and (3) functional requirements that continue to evolve.

Given these challenges, suppose the project's management team believes it has a 50% chance of completing the software development in accordance with the customer's planned schedule. From this, how might management use Bayes' rule to monitor whether the *chance* of completing the project on schedule is increasing or decreasing?

Mentioned above, *Bayesian inference* is a procedure that takes evidence, observations, or indicators as they emerge and applies Bayes' rule to infer the truthfulness or falsity of a hypothesis in terms of its probability. In this case, the hypothesis H is *Project XYZ will experience significant delays in completing its software development*.

Suppose at time t_1 the project's management comes to recognize that *project XYZ has been unable to fully staff to the number of software engineers needed for this effort*. In Bayesian inference, we treat this as an observation or evidence that has some bearing on the truthfulness of H. This is illustrated in Figure 2.3. Here, H is the hypothesis "node" and e_1 is the evidence node contributing to the truthfulness of H.

Given the evidence-to-hypothesis relationship in Figure 2.3 we can form the following equations from Bayes' rule.

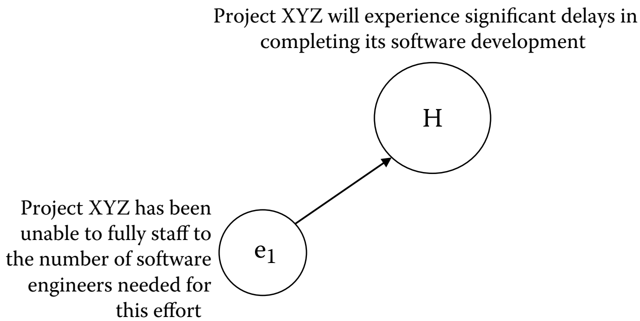


Figure 2.3: Evidence observed at time t_1.

$$P(H|e_1) = \frac{P(H)P(e_1|H)}{P(H)P(e_1|H) + P(H^c)P(e_1|H^c)} \tag{2.7}$$

$$P(H|e_1) = \frac{P(H)P(e_1|H)}{P(H)P(e_1|H) + (1 - P(H))P(e_1|H^c)} \tag{2.8}$$

Here, $P(H)$ is the team's initial or *prior* subjective (judgmental) probability that Project XYZ will be completed in accordance with the customer's planned schedule. Recall from the above discussion this was $P(H) = 0.50$. The other terms in Equation 2.7 (or Equation 2.8) are defined as follows: $P(H|e_1)$ is the probability H is true given evidence e_1, the term $P(e_1|H)$ is the probability evidence e_1 would be observed given H is *true*, and the term $P(e_1|H^c)$ is the probability evidence e_1 would be observed given H is *not true*.

Suppose this team's experience with e_1 is that staffing shortfalls is a factor that contributes to delays in completing software development projects. Given this, suppose they judge $P(e_1|H)$ and $P(e_1|H^c)$ to be 0.60 and 0.25, respectively.

From the evidence e_1 and the team's probability assessments related to e_1 we can compute a revised probability that Project XYZ will experience significant delays in completing its software development. This revised probability is given by Equation 2.9.

$$\begin{aligned} P(H|e_1) &= \frac{P(H)P(e_1|H)}{P(H)P(e_1|H) + (1 - P(H))P(e_1|H^c)} \\ &= \frac{(0.50)(0.60)}{(0.50)(0.60) + (1 - 0.50)(0.25)} = 0.70589 \end{aligned} \tag{2.9}$$

Notice the effect evidence e_1 has on increasing the probability that Project XYZ will experience a significant schedule delay. We've gone from the initial or *prior* probability of 50% to a *posterior* probability of just over 70%.

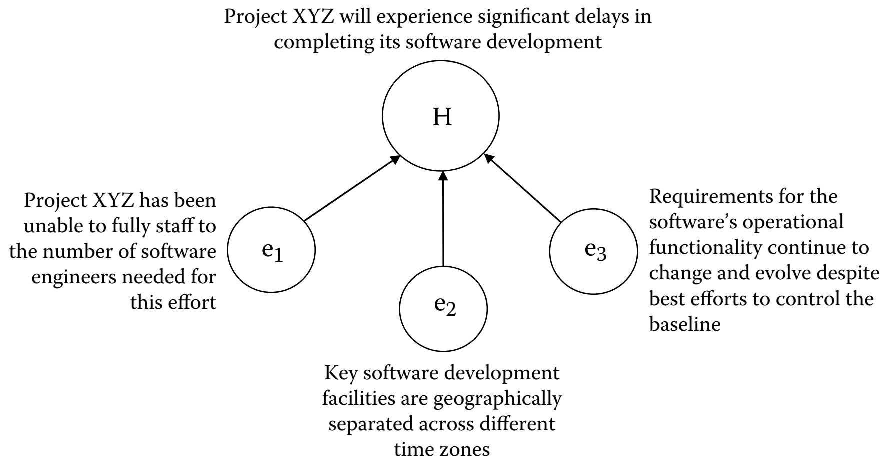


Figure 2.4: Evidence e_2 and e_3 observed at time t_2.

In the Bayesian inference community this is sometimes called *updating*; that is, updating the "belief" in the truthfulness of a hypothesis in light of observations or evidence that adds new information to the initial or prior assessments.

Next, suppose the management team observed two more evidence nodes at time t_2. Suppose these are in addition to the continued relevance of evidence node e_1. Suppose the nature of evidence nodes e_2 and e_3 are described in Figure 2.4. Now, what is the chance Project XYZ will experience a significant schedule delay given all the evidence collected in the set shown in Figure 2.4? Bayesian updating will again be used to answer this question.

Here, we will show how Bayesian updating is used to sequentially revise the *posterior* probability computed in Equation 2.9, to account for the observation of new evidence nodes e_2 and e_3. We begin by writing the following:

$$P(H\,|e_1 \cap e_2) \equiv P(H\,|e_1 e_2) \tag{2.10}$$

$$P(H\,|e_1 e_2) = \frac{P(H\,|e_1)\; P(e_2\,|H)}{P(H\,|e_1)\; P(e_2\,|H) + (1 - P(H\,|e_1))P(e_2\,|H^c)} \tag{2.11}$$

$$P(H\,|e_1 \cap e_2 \cap e_3) \equiv P(H\,|e_1 e_2 e_3) \tag{2.12}$$

$$P(H\,|e_1 e_2 e_3) = \frac{P(H\,|e_1 e_2)\; P(e_3\,|H)}{P(H\,|e_1 e_2)\; P(e_3\,|H) + (1 - P(H\,|e_1 e_2)\,|e_1))P(e_3\,|H^c)} \tag{2.13}$$

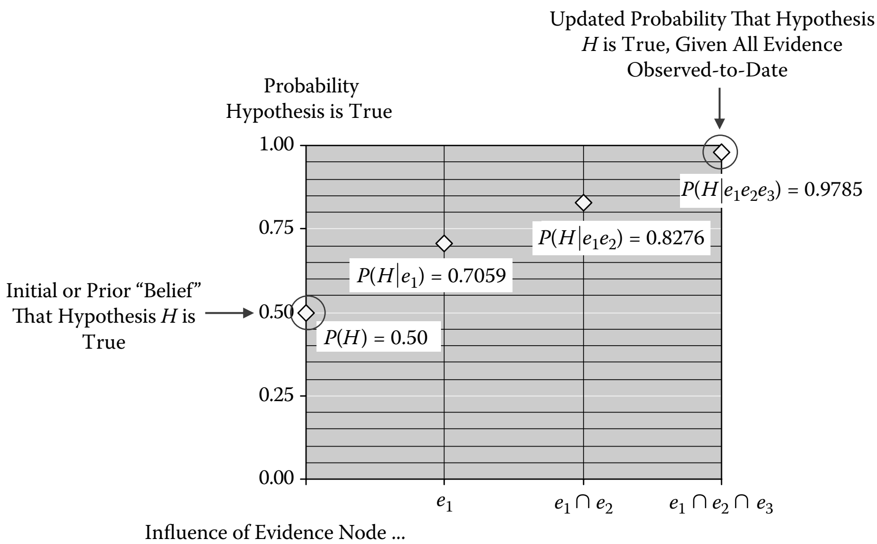


Figure 2.5: Bayesian updating: truthfulness of hypothesis H.

Suppose the management team made the following assessments.

$$P(e_2\,|H) = 0.90, \quad P(e_2\,|H^c) = 0.45$$
$$P(e_3\,|H) = 0.95, \quad P(e_3\,|H^c) = 0.10$$

Substituting them first into Equation 2.11 and then into 2.13 yields the following:

$$P(H|e_1e_2) = 0.8276 \quad \text{and} \quad P(H|e_1e_2e_3) = 0.9785$$

Thus, given the influence of *all* the evidence observed we can conclude hypothesis H is almost certain to occur. Figure 2.5 illustrates the findings from this analysis.

2.4.2 Writing a Risk Statement

Fundamentally, probability is a measure of the chance an *event* may or may not occur. Furthermore, all probabilities are conditional in the broadest sense that one can always write the following*:

$$Prob(A|\Omega) = Prob(A)$$

where A is an event (a subset) contained in the sample space Ω.

*This result derives from the fact that $Prob(\Omega|A) = 1$.

In a similar way, one can consider subjective or judgmental probabilities as conditional probabilities. The conditioning event (or events) may be experience with the occurrence of events known to have a bearing on the occurrence probability of the future event. Conditioning events can also manifest themselves as evidence, as discussed in the previous section on Bayesian inference.

Given these considerations, a "best practice" for expressing an identified risk is to write it in a form known as the *risk statement*. A risk statement aims to provide clarity and descriptive information about the identified risk so a reasoned and defensible assessment can be made on the risk's occurrence probability and its areas of impact or consequence (if the risk event occurs).

A protocol for writing a risk statement is the *Condition-If-Then* construct. This protocol applies in all risk management processes designed for any systems engineering environment. It is a recognition that a risk event is, by its nature, a probabilistic event and one that, if it occurs, has unwanted consequences.

What is the *Condition-If-Then* construct? The *Condition* reflects what is known today. It is the *root cause* of the identified risk event. Thus, the *Condition* is an event that has occurred, is presently occurring, or will occur with certainty. Risk events are future events that may occur *because* of the *Condition* present. Below is an illustration of this protocol.

Suppose we have the following two events. Define the *Condition* as event B and the *If* as event A (the risk event)

$$\begin{aligned} B = \{&\text{Current test plans are focused on the components of the subsystem} \\ &\text{and not on the subsystem as a whole}\} \\ A = \{&\text{Subsystem will not be fully tested when integrated into the system for} \\ &\text{full-up system-level testing}\} \end{aligned}$$

The risk event is the *Condition-If* part of the construct; specifically,

> **Risk Event:** {*The subsystem will not be fully tested when integrated into the system for full-up system-level testing, because current test plans are focused on the components of the subsystem and not on the subsystem as a whole.*}

From this, we see the *Condition-If* part of the risk statement construct is equivalent to a probability event; formally, we can write

$$0 < P(A \mid B) = \alpha < 1$$

where α is the probability risk event A occurs given the conditioning event B (the root cause event) has occurred. Why do you think $P(A \mid B)$ here is written as a strict inequality?

In the above, it was explained why a risk event is equivalent to a probability event; that is, the *Condition-If* part of the risk statement construct. The *Then* part of the construct contains additional information; that is, information on the risk's consequences. An example of a risk statement is shown in Figure 2.6.

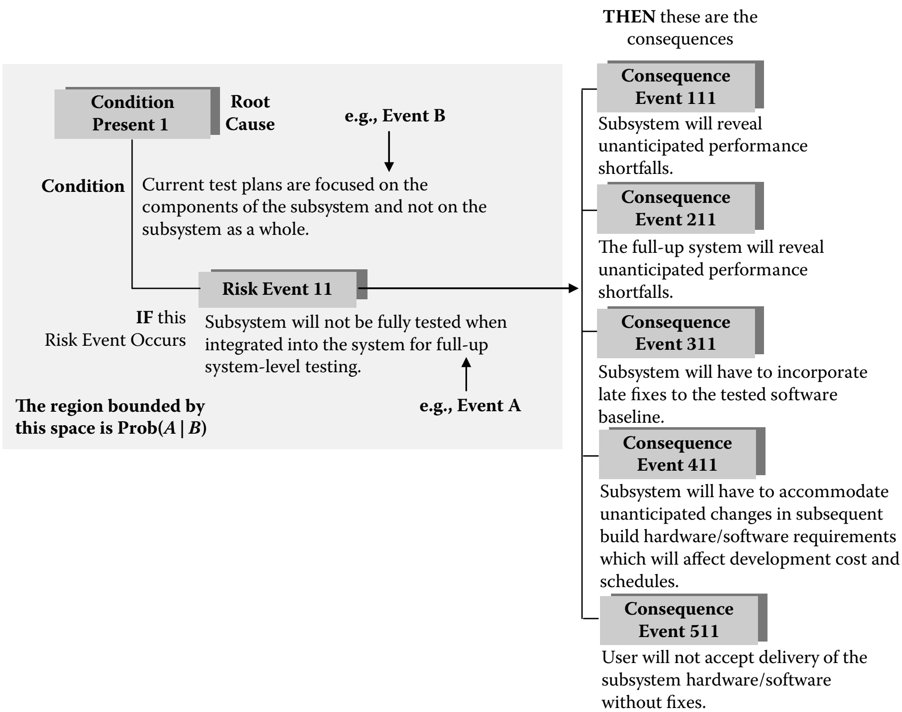


Figure 2.6: The Risk Statement: An Illustration of the *Condition-If-Then* construct [4].

Questions and Exercises

1. State the interpretation of probability implied by the following:

   (a) The probability a tail appears on the toss of a fair coin is 1/2.
   (b) After recording the outcomes of 50 tosses of a fair coin, the probability a tail appears is 0.54.
   (c) It is with certainty the coin is fair.
   (d) The probability is 60% that the stock market will close 500 points above yesterday's closing count.
   (e) The design team believes there is less than a 5% chance the new microchip will require more than 12,000 gates.

2. A sack contains 20 marbles exactly alike in size but different in color. Suppose the sack contains 5 blue marbles, 3 green marbles, 7 red marbles, 2 yellow marbles, and 3 black marbles. Picking a single marble from the sack and then replacing it, what is the probability of choosing the following:

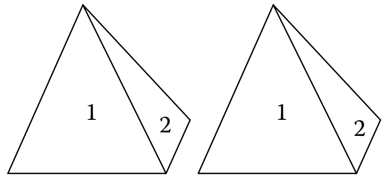
   (a) Blue marble? (b) Green marble? (c) Red marble?
   (d) Yellow marble? (e) Black marble? (f) Non-blue marble
   (g) Red or non-red marble?

3. If a fair coin is tossed, what is the probability of not obtaining a head? What is the probability of the event: (a head or not a head)?

4. Suppose A is an event (a subset) contained in the sample space Ω. Given this, are the following probability statements true or false, and why?

   (a) $P(A \cup A^c) = 1$ (b) $P(A \mid \Omega) = P(A)$

5. Suppose two tetrahedrons (4-sided polygons) are randomly tossed. Assuming the tetrahedrons are weighted fair, determine the set of all possible outcomes Ω. Assume each face is numbered 1, 2, 3, and 4.



Two tetrahedrons for Exercise 5.

Let the sets A, B, C, and D represent the following events

A: The sum of the toss is even

B: The sum of the toss is odd

C: The sum of the toss is a number less than 6

D: The toss yielded the same number on each upturned face

(a) Find $P(A)$, $P(B)$, $P(C)$, $P(A \cap B)$, $P(A \cup B)$, $P(B \cup C)$, and $P(B \cap C \cap D)$.

(b) Verify $P((A \cup B)^c) = P(A^c \cap B^c)$.

6. The XYZ Corporation has offers on two contracts A and B. Suppose the proposal team made the following subjective probability assessments: the chance of winning contract A is 40%, the chance of winning contract B is 20%, the chance of winning contract A or contract B is 60%, the chance of winning both contracts is 10%.

   (a) Explain why the above set of probability assignments is *inconsistent* with the axioms of probability.

   (b) What must $P(B)$ equal such that it and the set of other assigned probabilities specified above are consistent with the axioms of probability?

7. Suppose a coin is balanced such that tails appears three times more frequently than heads. Show the probability of obtaining a tail with such a coin is 3/4. What would you expect this probability to be if the coin was fair — that is, equally balanced?

8. Suppose the sample space of an experiment is given by $\Omega = A \cup B$. Compute $P(A \cap B)$ if $P(A) = 0.25$ and $P(B) = 0.80$.

9. If A and B are disjoint subsets of Ω show that

   (a) $P(A^c \cup B^c) = 1$

   (b) $P(A^c \cap B^c) = 1 - [P(A) + P(B)]$

10. Two missiles are launched. Suppose there is a 75% chance missile A hits the target and a 90% chance missile B hits the target. If the probability missile A hits the target is *independent* of the probability missile B hits the target, determine the probability missile A or missile B hits the target. Find the probability needed for missile A such that if the probability of missile B hitting the target remains at 90%, the probability missile A or missile B hits the target is 0.99.

11. Suppose A and B are independent events. Show that
    (a) The events A^c and B^c are independent.
    (b) The events A and B^c are independent.
    (c) The events A^c and B are independent.

12. Suppose A and B are independent events with $P(A) = 0.25$ and $P(B) = 0.55$. Determine the probability
    (a) At least one event occurs.
    (b) Event B occurs but event A does not occur.

13. Suppose A and B are independent events with $P(A) = r$ and the probability that at least A or B occurs is s. Show the only value for $P(B)$ is the product $(s - r)(1 - r)^{-1}$.

14. At a local sweet shop, 10% of all customers buy ice cream, 2% buy fudge, and 1% buy both ice cream and fudge. If a customer selected at random bought fudge, what is the probability the customer bought an ice cream? If a customer selected at random bought ice cream, what is the probability the customer bought fudge?

15. For any two events A and B, show that $P(A \,|\, A \cap (A \cap B)) = 1$.

16. A production lot contains 1000 microchips of which 10% are defective. Two chips are successively drawn at random without replacement. Determine the probability
    (a) Both chips selected are non-defective.
    (b) Both chips are defective.
    (c) The first chip is defective and the second chip is non-defective.
    (d) The first chip is non-defective and the second chip is defective.

17. Suppose the sampling scheme in exercise 16 was with replacement, that is, the first chip is returned to the lot before the second chip is drawn. Show how the probabilities computed in exercise 16 change.

18. Spare power supply units for a communications terminal are provided to the government from three different suppliers A_1, A_2, and A_3. Suppose 30% come from A_1, 20% come from A_2, and 50% come from A_3. Suppose these units occasionally fail to perform according to their specifications and the following has been observed: 2% of those supplied by A_1 fail, 5% of those supplied by A_2 fail, and 3% of those supplied by A_3 fail. What is the probability any one of these units provided to the government will perform *without* failure?

19. In a single day, ChipyTech Corporation's manufacturing facility produces 10,000 microchips. Suppose machines A, B, and C individually produce 3000, 2500, and 4500 chips daily. The quality control group has determined the output from machine A has yielded 35 defective chips, the output from machine B has yielded 26 defective chips, and the output from machine C has yielded 47 defective chips.
    (a) If a chip was selected at random from the daily output, what is the probability it is defective?
    (b) What is the probability a randomly selected chip was produced by machine A? By machine B? By machine C?
    (c) Suppose a chip *was* randomly selected from the day's production of 10,000 microchips and it was found to be defective. What is the probability it was produced by machine A? By machine B? By machine C?

20. From section 2.4.1, show that Bayes' rule is the basis for the equations below.

(a) $P(H\,|e_1) = \dfrac{P(H)P(e_1\,|H)}{P(H)P(e_1\,|H) + (1 - P(H))P(e_1\,|H^c)}$

(b) $P(H\,|e_1e_2) = \dfrac{P(H\,|e_1)\; P(e_2\,|H)}{P(H\,|e_1)\; P(e_2\,|H) + (1 - P(H\,|e_1))P(e_2\,|H^c)}$

(c) $P(H\,|e_1e_2e_3) = \dfrac{P(H\,|e_1e_2)\; P(e_3\,|H)}{P(H\,|e_1e_2)\; P(e_3\,|H) + (1 - P(H\,|e_1e_2)\;|e_1))P(e_3\,|H^c)}$

References

[1] Feller, W. 1968. *An Introduction to Probability Theory and Its Applications*, vol. 1, 3rd ed (revised). New York: John Wiley & Sons, Inc.

[2] Hitch, C. J. 1955. An Appreciation of Systems Analysis, P-699. Santa Monica, California: The RAND Corporation.

[3] Garvey, P. R., 2000. *Probability Methods for Cost Uncertainty Analysis: A Systems Engineering Perspective*, New York: Marcel Dekker.

[4] Garvey, P. R., January 2005. "System-of-Systems Risk Management: Perspectives on Emerging Process and Practice," The MITRE Corporation, MP 04B0000054.

Chapter 3

Elements of Decision Analysis

3.1 Introduction

Many decisions involve choosing the "best" or "most preferred" option among a set of competing options. In this chapter, we touch on the field of decision analysis and discuss elements of this subject designed to identify not only the best option but an ordering of options from most-to least-preferred, as a function of how well each option performs against a set of evaluation criteria.

In engineering risk management, decision-makers need to order risks from most- to least-critical for a variety of purposes. A primary one is to decide where risk mitigation resources should be allocated. In this context, risks are analogous to options. Their criticality is a function of multiple evaluation criteria, such as a risk's impact on a system's cost, schedule, or technical performance.

Although modern decision analysis has much of its theoretical basis in *Decisions With Multiple Objectives: Preferences and Value Tradeoffs*, by R. L. Keeney and H. Raiffa [1], eighteenth century mathematicians Daniel Bernoulli and Gabriel Cramer significantly contributed to its early development. This chapter will explore elements and concepts of modern decision analysis and illustrate their application from an engineering risk management perspective.

3.2 The Value Function

This section introduces the concepts of value, utility, and risk functions. These concepts form the analytical basis for applying decision analysis to problems of ranking and identifying the most preferred option among a set of competing options.

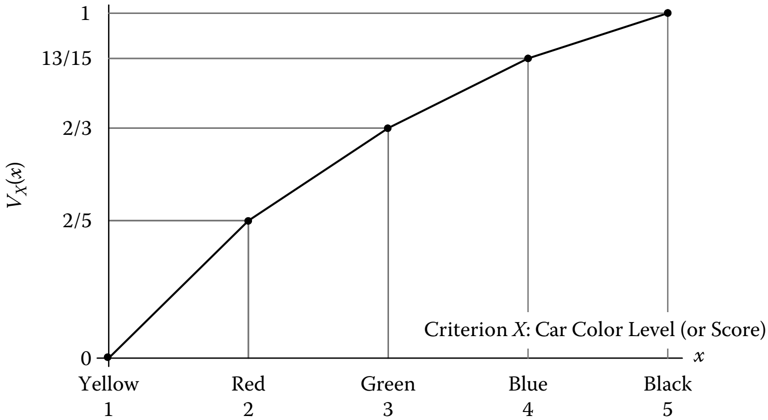


Figure 3.1: An illustrative value function.

A *value function* is a real-valued mathematical function defined over an evaluation criterion (or attribute) that represents an option's measure of "goodness" over the levels of the criterion. A measure of goodness reflects a decision-maker's judged value in the performance of an option across the levels of a criterion (or attribute).

A value function is usually designed to vary from zero to one over the range of levels (or scores) for a criterion. In practice, the value function for a criterion's least preferred level (or score) (i.e., the least preferred option or alternative) takes the value zero. The value function for a criterion's most preferred level (or score) (i.e., the most preferred option or alternative) takes the value one.

Figure 3.1 illustrates a buyer's value function for the criterion *Car Color*. A value function such as the one shown in Figure 3.1 is known as a *single dimensional value function* (SDVF) or a *single attribute value function* (SAVF).

A notation in Figure 3.1 operates as follows. The letter capital X denotes the criterion *Car Color*. The lower-case x denotes the level (or score) for a specific option or alternative associated with the criterion X. The notation $V_X(x)$ denotes the value of x. For example, for the criterion X = Car Color, the option (or alternative) x = Green = 3 has a value of 2/3; that is,

$$V_X(\text{Green}) = V_{Car\ Color}(3) = \frac{2}{3} \tag{3.1}$$

In Figure 3.1, suppose a buyer has the following preferences for the criterion *Car Color*. Yellow is the least preferred color while black is the most preferred color on this criterion. These colors receive a value of zero and one, respectively. Furthermore, the value function in Figure 3.1 shows the buyer's increasing value of color as the level (or score) of the criterion moves from the color yellow to the color black. Here, red is preferred to yellow; green is preferred to red; blue is preferred to green; black is preferred to blue.

In Figure 3.1, the values show not only an ordering of preferences but suppose the buyer's strength of preference for one color over another is also captured. Here, the smallest increment (change) in value occurs between the color blue and the color black. If we use this increment as a reference standard, then it can be shown that, for this buyer, the value increment between yellow and red is three times the value increment between blue and black; the value increment between red and green is two times the value increment between blue and black; the value increment between green and blue is one and a half times the value increment between blue and black.

The expression "value increment" or "increment in value" refers to the degree to which the buyer, in this case, prefers the higher level (score) to the lower level (score) [2]. In Figure 3.1, the value increment between yellow and red is greater than the value increment between blue and black. Thus, increasing from yellow to red is more preferable, for this buyer, than increasing from blue to black.

Because the buyer's value function in Figure 3.1 features a preference ordering and a strength of preference between the criterion's levels (or scores), this function is known as a *measurable value function*. In a measurable value function the value difference between any two levels (or scores) within a criterion (or attribute) represents a decision-maker's strength of preference between the two levels (or scores). The vertical axis of a measurable value function is a *cardinal interval scale*[*] measure of the strength of a decision-maker's preferences. For this reason, a measurable value function is also referred to as a *cardinal value function*. Refer to Kirkwood [2] and Dyer and Sarin [3] for a further technical discussion on measurable value functions.

Figure 3.1 also illustrates how a value function can combine both cardinal and ordinal features. In this case, the vertical axis is a cardinal interval scale whereas the

---

[*] A further discussion of interval scales, and the other classical measurement scales, is provided later in this section.

horizontal axis is an ordinal scale. In Figure 3.1, the values along the vertical axis have, to the decision-maker, meaningful preference differences between them. The horizontal axis is ordinal in the sense that red is preferred to yellow; green is preferred to red; blue is preferred to green; black is preferred to blue. Here, we have an ordering of preference only that is preserved. The distance between colors along the horizontal axis is indeterminate (i.e., not meaningful).

Developing a Piecewise Linear Single Dimensional Value Function

The value function in Figure 3.1 is known as a *piecewise linear single dimensional value function*. This function is made up of four individual line segments joined together at their "value points." Piecewise linear single dimensional value functions are commonly developed in cases when only a few levels (or scores) define a criterion, such as *Car Color*.

Value Increment Approach

One procedure for developing a piecewise linear single dimensional value function is the *value increment approach*, described and illustrated by Kirkwood [2]. This approach requires increments of value be specified between a criterion's levels (or scores). Furthermore, the sum of these value increments from the lowest level (or score) to the highest level (or score) is one [2].

In Figure 3.1, the value increments from the lowest level (or score) to the highest level (or score) are, respectively,

$$\left\{\frac{2}{5}, \frac{4}{15}, \frac{3}{15}, \frac{2}{15}\right\} = \left\{\frac{6}{15}, \frac{4}{15}, \frac{3}{15}, \frac{2}{15}\right\}$$

Note their sum is one and 2/15 is the smallest value increment. That is, for this buyer, the smallest value increment is between blue and black. A generalization of this is shown in Figure 3.2.

In Figure 3.2, the smallest value increment for criterion X is between levels (or scores) A4 and A5 and is denoted by Δ. Subsequent value increments are multiples of the smallest value increment; that is, $a\Delta$, $b\Delta$, and $c\Delta$, where a, b, and c are positive constants. Mentioned previously, it follows that

$$c\Delta + b\Delta + a\Delta + \Delta = 1 \tag{3.2}$$

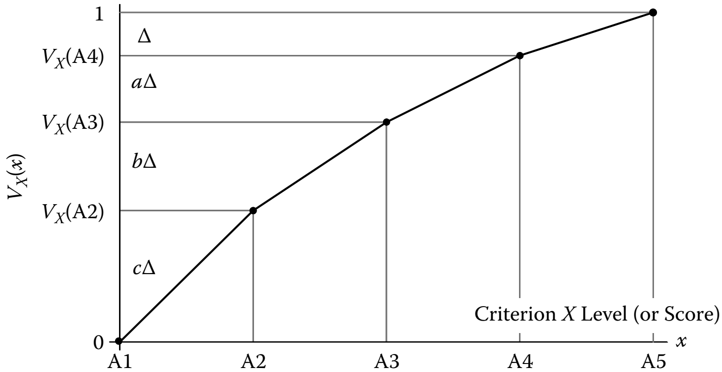


Figure 3.2: A piecewise linear value function.

Furthermore, the following equations are true

$$V_X(\text{A1}) = 0$$
$$V_X(\text{A2}) = V_X(\text{A1}) + c\Delta = c\Delta$$
$$V_X(\text{A3}) = V_X(\text{A2}) + b\Delta = c\Delta + b\Delta$$
$$V_X(\text{A4}) = V_X(\text{A3}) + a\Delta = c\Delta + b\Delta + a\Delta$$
$$V_X(\text{A5}) = V_X(\text{A4}) + \Delta = c\Delta + b\Delta + a\Delta + \Delta = 1$$

This approach can be related to the value function for *Car Color*, shown in Figure 3.1. Here, the value increments for the different levels (or scores) for car color, as multiples of the smallest value increment Δ, is shown in Figure 3.3.

In Figure 3.3, the smallest increment in value Δ occurs between the color blue and the color black. If we use Δ as the reference standard, then it can be seen in Figure 3.3 that, for this buyer, the value increment between yellow and red is three times the smallest value increment Δ; the value increment between red and green is two times the smallest value increment Δ; the value increment between green and blue is one and a half times the smallest value increment Δ.

From this, it follows that

$$V_X(\text{Yellow}) = 0$$
$$V_X(\text{Red}) = V_X(\text{Yellow}) + 3\Delta = 0 + 3\Delta = \frac{6}{15} = \frac{2}{5}$$

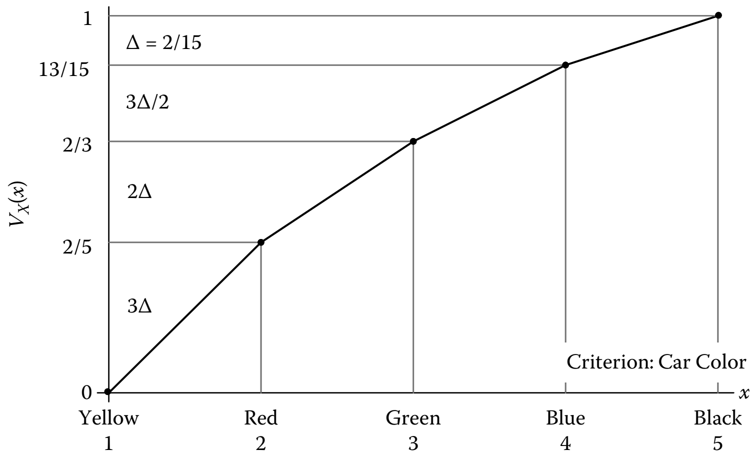


Figure 3.3: A value function for *Car Color*: a value increment view.

$$V_X(\text{Green}) = V_X(\text{Red}) + 2\Delta = \frac{2}{5} + \frac{4}{15} = \frac{2}{3}$$
$$V_X(\text{Blue}) = V_X(\text{Green}) + \frac{3}{2}\Delta = \frac{2}{3} + \frac{3}{2}\frac{2}{15} = \frac{13}{15}$$
$$V_X(\text{Black}) = V_X(\text{Blue}) + \Delta = \frac{13}{15} + \frac{2}{15} = 1$$

Direct Preference Rating Approach

Another approach to specifying a single dimensional value function is the direct subjective assessment of value. This is sometimes referred to as *direct rating*. Here, the value function for a criterion's option (or alternative) with the least preferred level (or score) is assigned the value zero. The value function for a criterion's option (or alternative) with the most preferred level (or score) is assigned the value one.

Next, the intermediate options, or alternatives, are ranked such that their ranking reflects a preference ordering along the horizontal axis of the value function. With this, the values of these intermediate options (or alternatives) are directly assessed such that they fall between zero and one along the vertical axis of the value function. The spacing (or distance) between the values of these intermediate options (or alternatives) is intended to reflect the strength of preference of the expert (or team) making the assessments for one option (or alternative) over another.

Because values are directly assessed along an interval scale it is important to check for consistency. As will be discussed later in this section, *differences in values* along an *interval scale have meaning*. For example, a value difference of 0.30 points between two options (or alternatives) should reflect an improvement in value that is exactly twice that measured by a difference of 0.15 points between two other options (or alternatives).

When implementing the direct preference rating approach it is important to check for bias and dominance of opinion by a single expert or decision-maker. This is a potential issue that has to be managed in the process of eliciting such judgmental values.

The Exponential Value Function

A special type of value function, known as the *exponential value function*, is sometimes used as an alternative to developing a piecewise linear single dimensional value function. Kirkwood [2] has developed and written extensively on the theory and application of the exponential value function and has provided a number of examples of its use.

In general, the exponential value function can be used to represent increasing or decreasing values (preferences) for criteria characterized by a continuous range of levels (or scores). For example, the criterion *Probability of Intercept* might have the value function shown in Figure 3.4.

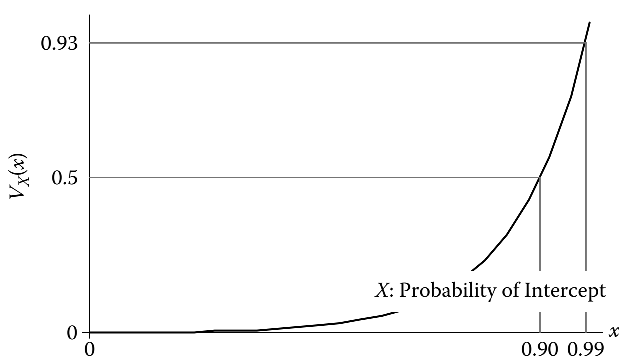


Figure 3.4: An exponential value function for *probability of intercept*.

Here, higher probabilities of a successful intercept are more valued than lower probabilities. Furthermore, the level (scores) for this criterion vary continuously across the range of probability; that is, between zero and one along the horizontal axis.

Definition 3.1 If values (preferences) are monotonically increasing over the levels (scores) for an evaluation criterion X, then the exponential value function is given by

$$V_X(x) = \begin{cases} \dfrac{1 - e^{-(x - x_{\min})/\rho}}{1 - e^{-(x_{\max} - x_{\min})/\rho}} & \text{if } \rho \neq \infty \\ \dfrac{x - x_{\min}}{x_{\max} - x_{\min}} & \text{if } \rho = \infty \end{cases} \tag{3.3}$$

Definition 3.2 If values (preferences) are monotonically decreasing over the levels (scores) for an evaluation criterion X, then the exponential value function is given by

$$V_X(x) = \begin{cases} \dfrac{1 - e^{-(x_{\max} - x)/\rho}}{1 - e^{-(x_{\max} - x_{\min})/\rho}} & \text{if } \rho \neq \infty \\ \dfrac{x_{\max} - x}{x_{\max} - x_{\min}} & \text{if } \rho = \infty \end{cases} \tag{3.4}$$

A family of exponential value functions is shown in Figure 3.5. The left-most picture reflects exponential value functions for monotonically increasing preferences over the criterion X. The right-most picture reflects exponential value functions for monotonically decreasing preferences over the criterion X.

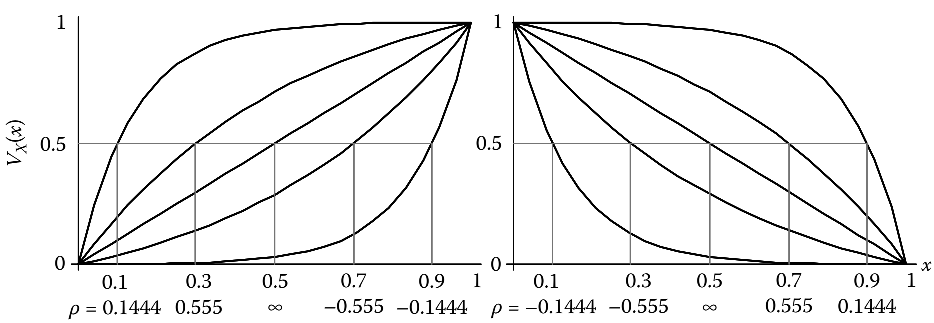


Figure 3.5: Families of exponential value functions.

Working with the Exponential Value Function

The shape of the exponential value function is governed by the parameter ρ, referred to as the *exponential constant* [2]. One procedure for determining ρ relies on identifying the midvalue associated with the range of levels (or scores) for the evaluation criterion of interest.

Definition 3.3 Midvalue [2] The midvalue of a criterion X over a range of possible levels (scores) for X is defined to be the level (score) x such that the difference in *value* between the lowest level (score) x_{min} and the midvalue x_{mid} is the same as the difference in *value* between the midvalue x_{mid} and the highest level (score) x_{max} [2].

From this definition, it follows that the single dimensional value for the midvalue x_{mid} of X will always equal 0.5; that is, $V_X(x_{\text{mid}}) = 0.5$. If x_{min}, x_{max}, and x_{mid} are known (or given), then Equation 3.3 or Equation 3.4 can be numerically solved for ρ.

For example, in Figure 3.4, suppose the midvalue x_{mid} for the criterion *Probability of Intercept* was assessed to be 0.90. Since this criterion is characterized by increasing preferences, Equation 3.3 is the appropriate form of the exponential value function. To determine ρ, in this case, we need to solve the following:

$$V_X(0.90) = 0.5 = \frac{1 - e^{-(0.90-0)/\rho}}{1 - e^{-(1-0)/\rho}} = \frac{1 - e^{-(0.90)/\rho}}{1 - e^{-(1)/\rho}} \tag{3.5}$$

Solving Equation 3.5 numerically yields $\rho = -0.1444475$. To solve Equation 3.5, a number of software applications are available such as Microsoft's Excel® Goal Seek or Solver routines. Here, an application known as *Mathematica*® [4] was used. The specific routine is

FindRoot $[(1 - \text{Exp}[-0.9/\rho])/(1 - \text{Exp}[-1/\rho]) == 0.5, \{\rho, -1\}]$

which, in *Mathematica*®, returns the value $\rho = -0.144475$.

The following illustrates an exponential value function for monotonically decreasing preferences. Suppose the criterion *Repair Time* for a mechanical device, measured in hours, ranges from 10 to 30 hours. Suppose the midvalue for this criterion was assessed at 23 hours. We need to determine the exponential constant ρ for this exponential value function, depicted in Figure 3.6.

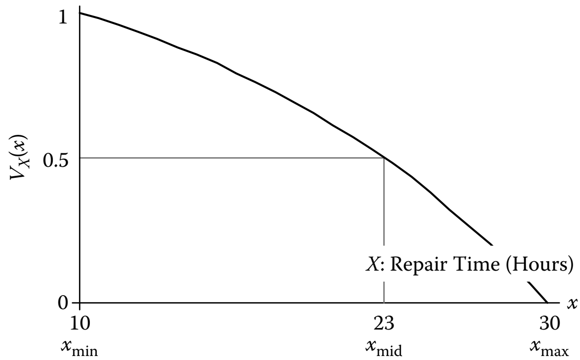


Figure 3.6: An exponential value function for Repair Time.

Since values (preferences) are monotonically decreasing over the levels (scores) for the evaluation criterion X, *Repair Time*, the exponential value function is given by Equation 3.4; that is:

$$V_X(x) = \frac{1 - e^{-(x_{max}-x)/\rho}}{1 - e^{-(x_{max}-x_{min})/\rho}}$$

which, for this case, is

$$V_X(x) = \frac{1 - e^{-(30-x)/\rho}}{1 - e^{-(30-10)/\rho}} = \frac{1 - e^{-(30-x)/\rho}}{1 - e^{-(20)/\rho}}$$

Since $x_{mid} = 23$, we have

$$V_X(23) = 0.5 = \frac{1 - e^{-(30-23)/\rho}}{1 - e^{-(20)/\rho}} = \frac{1 - e^{-(7)/\rho}}{1 - e^{-(20)/\rho}} \tag{3.6}$$

Solving Equation 3.6 numerically yields $\rho = 15.6415$. Again, this was done using the *Mathematica*® routine:

FindRoot [(1 – Exp[−7/ρ])/(1 – Exp[−20/ρ]) == 0.5, {ρ, 1}]

which, in *Mathematica*®, returns the value $\rho = 15.6415$.

In the discussion above, the exponential value function's exponential constant ρ was determined by setting $V_X(x_{mid})$ equal to 0.5 and then solving for ρ numerically. That is, for a given criterion X, its score (level) x is assessed such that it represents the midvalue of the value function $V_X(x)$. In practice, one is not restricted to solving ρ based on the midvalue. It might happen an evaluator is better able to assess the level (score) x associated with a value function's value of 0.25 or 0.75. In such cases, a similar procedure applies with respect to solving for ρ.

Figure 3.5 illustrated a family of exponential value functions based on assessments of midvalues. Figure 3.7 illustrates the same family of exponential value functions based on assessments of "quarter-values." Here, the level (score) x of criterion X is assessed such that it represents a value of 0.25 for the value function $V_X(x)$.

In Figure 3.7, the left-most picture reflects exponential value functions for monotonically increasing preferences over the criterion X. This picture is a plot of Equation 3.3; specifically,

$$V_X(x) = \begin{cases} \dfrac{1 - e^{-(x-0)/\rho}}{1 - e^{-(1-0)/\rho}} & \text{if } \rho \neq \infty \\ \dfrac{x - 0}{1 - 0} & \text{if } \rho = \infty \end{cases} = \begin{cases} \dfrac{1 - e^{-(x)/\rho}}{1 - e^{-(1)/\rho}} & \text{if } \rho \neq \infty \\ x & \text{if } \rho = \infty \end{cases} \tag{3.7}$$

Referring to the left-most picture of Figure 3.7 $x = 0.7$ is associated with a value of 0.25 for the value function $V_X(x)$. From Equation 3.7 it follows that:

$$V_X(0.7) = 0.25 = \frac{1 - e^{-(0.7)/\rho}}{1 - e^{-(1)/\rho}} \tag{3.8}$$

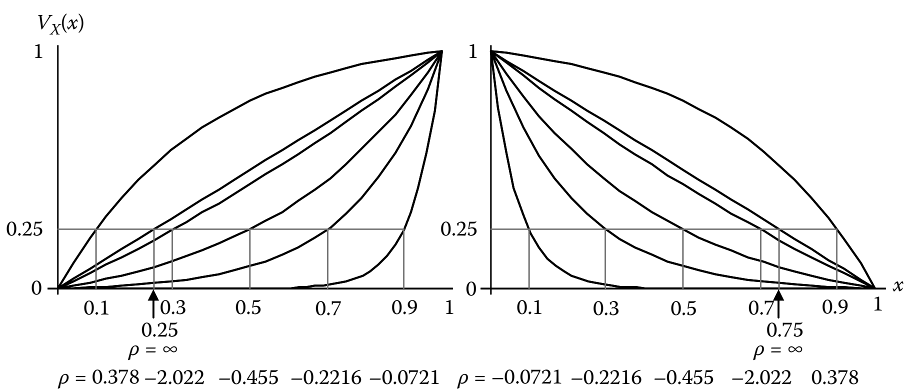


Figure 3.7: Families of exponential value functions, quarter-value based.

Solving Equation 3.8 numerically yields $\rho = -0.2216$. Mentioned previously, this was done using the *Mathematica*® routine:

$$\textbf{FindRoot}\ [(1 - \text{Exp}[-0.7/\rho])/(1 - \text{Exp}[-1/\rho]) == 0.25, \{\rho, -1\}]$$

The Additive Value Function

Deciding on the "best" alternative from a number of competing alternatives is often based on their performance across n evaluation criteria. When n criteria are involved, a set of n value functions is usually defined over each criterion. Given a set of n value functions defined over n criteria what is an alternative's overall value across these criteria? Some definitions are needed before answering this question.

Definition 3.4 *Preferential Independence:* A criterion Y is *preferentially independent* of another criterion X if preferences for particular outcomes of Y do not depend on the level (or score) of criterion X.

Informally, preference independence is present if a decision-maker's preference ranking for one criterion (or attribute) does not depend on *fixed* levels (or scores) of other criteria (or attributes) in the decision space.

To illustrate preferential independence, consider a buyer's selection preference for a new car. If criterion Y is price and criterion X is color, then price Y is preferentially independent of color X if the buyer prefers a lower price to a higher price regardless of the car's color.

Definition 3.5 *Mutual Preferential Independence:* If an evaluator's preference for the ith criterion (in a set of n criteria) remains the same regardless of the level (or score) of the other criteria, then the ith criterion is preferentially independent of the other criteria. If each criterion is preferentially independent of the other criteria, then the entire set of criteria is called *mutually preferentially independent*.

Definition 3.6 *Additive Value Function:* A value function $V_Y(y)$ is an additive value function if there exists n single dimensional value functions $V_{X_1}(x_1)$, $V_{X_2}(x_2)$, $V_{X_3}(x_3)$, ... , $V_{X_n}(x_n)$ satisfying

$$V_Y(y) = w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + w_3 V_{X_3}(x_3) + \cdots + w_n V_{X_n}(x_n)$$

where w_i for $i = 1, \ldots, n$ are non-negative weights (importance weights) whose values range between zero and one and where $w_1 + w_2 + w_3 + \cdots + w_n = 1$.

Theorem 3.1 *If the set of criteria is mutually preferentially independent, then the evaluator's preferences can be represented by an additive value function.*

A proof of Theorem 3.1 is outside the scope of this text; however, the reader is directed to Keeney and Raiffa [1] for a proof of this theorem.

Given the conventions that (1) the single dimensional value functions $V_{X_1}(x_1)$, $V_{X_2}(x_2)$, $V_{X_3}(x_3)$, ..., $V_{X_n}(x_n)$ each range in value between zero and one and (2) the weights each range in value between zero and one and sum to unity it follows that $V_Y(y)$ will range between zero and one. Thus, the higher the value of $V_Y(y)$ the more preferred the alternative; similarly, the lower the value of $V_Y(y)$ the less preferred the alternative.

Case Discussion 3.1 Consider the following case. Suppose a buyer needs to identify which car option (of five options being considered) is "best" across three evaluation criteria: *Car Color*, *Miles per Gallon*, and *Price*. Furthermore, suppose the criterion *Miles per Gallon* is twice as important as the criterion *Car Color* and *Car Color* is half as important as *Price*. Suppose the buyer made the value assessments in Figure 3.8 for each criterion. Assume these criteria are mutually preferentially independent.

A Solution to Case Discussion 3.1 Mentioned above, the three criteria are assumed to be mutually preferentially independent. It follows that the additive value function can be used to generate an overall score for the performance of each car across the three evaluation criteria. In this case, the additive value function is

$$V_Y(y) = w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + w_3 V_{X_3}(x_3) \tag{3.9}$$

where $V_{X_1}(x_1)$, $V_{X_2}(x_2)$, and $V_{X_3}(x_3)$ are the value functions for *Car Color*, *Miles per Gallon*, and *Price*, respectively; and, w_i for $i = 1, 2, 3$ are non-negative weights (importance weights) whose values range between zero and one and where $w_1 + w_2 + w_3 = 1$.

Weight Determination

Since the criterion *Miles per Gallon* was given to be twice as important as the criterion *Car Color* and *Car Color* was given to be half as important as *Price,* the weights in Equation 3.9 are determined as follows:

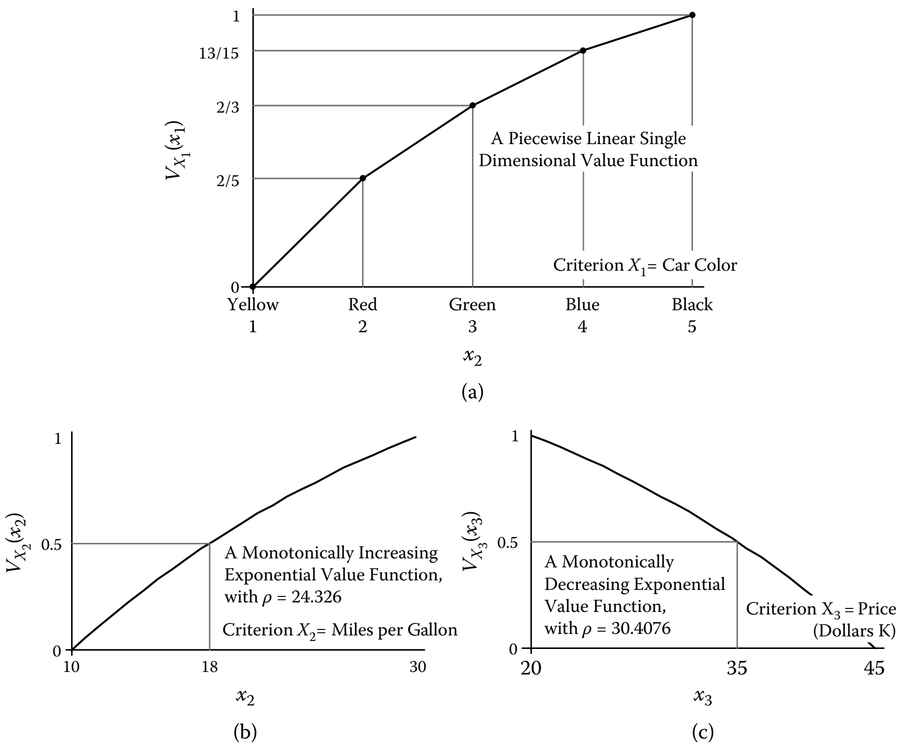


Figure 3.8: Single dimensional value functions for Case Discussion 3.1.

Let w_1 denote the weight for *Car Color*, w_2 denote the weight for *Miles per Gallon*, and w_3 denote the weight for *Price*. It follows that $w_2 = 2w_1$, $w_1 = \frac{1}{2}w_3$, and this implies $w_2 = 2(\frac{1}{2}w_3) = w_3$.

Since $w_1 + w_2 + w_3 = 1$ it follows that

$$\frac{1}{2}w_3 + w_3 + w_3 = 1 \Rightarrow \frac{5}{2}w_3 = 1 \Rightarrow w_3 = \frac{2}{5} \Rightarrow w_2 = \frac{2}{5} \Rightarrow w_1 = \frac{1}{2}w_3 = \frac{1}{5}$$

From this, Equation 3.9 can be written as

$$V_Y(y) = \frac{1}{5}V_{X_1}(x_1) + \frac{2}{5}V_{X_2}(x_2) + \frac{2}{5}V_{X_3}(x_3) \tag{3.10}$$

Determine Performance Matrix

Next, suppose the buyer collected data on the five car options according to their performance against each of the three criteria *Car Color*, *Miles per Gallon*,

TABLE 3.1: A Performance Matrix of the Buyer's Car Options

Criterion Level (Score)	**Color**	**MPG**	**Price**	**Equivalent Value Scores**	**Color**	**MPG**	**Price**	**Overall Value Score**
Car 1	4	15	30	Car 1	0.87	0.33	0.69	0.58
Car 2	1	22	25	Car 2	0.00	0.69	0.86	0.62
Car 3	5	18	38	Car 3	1.00	0.50	0.37	0.55
Car 4	3	12	42	Car 4	0.67	0.14	0.17	0.26
Car 5	2	28	21	Car 5	0.40	0.93	0.97	0.84

and *Price*. In Table 3.1, the left half of the matrix shows the raw data on each car option across these criteria. The right half of the matrix shows the raw data as scores from the value functions in Figure 3.8.

The overall value scores in Table 3.1 derive from Equation 3.10; that is,

$$V_Y(y) = \frac{1}{5}V_{X_1}(x_1) + \frac{2}{5}V_{X_2}(x_2) + \frac{2}{5}V_{X_3}(x_3)$$

From Figure 3.8, and Equation 3.3 and Equation 3.4, respectively, we have

$$V_{X_2}(x_2) = \frac{1 - e^{-(x_2-10)/24.326}}{1 - e^{-(30-10)/24.326}}$$

and

$$V_{X_3}(x_3) = \frac{1 - e^{-(45-x_3)/30.4076}}{1 - e^{-(45-20)/30.4076}}$$

For example, Car 1 has the following overall value score.

$$V_Y(y) = \frac{1}{5}V_{X_1}(4) + \frac{2}{5}V_{X_2}(15) + \frac{2}{5}V_{X_3}(30)$$

$$V_Y(y) = \frac{1}{5}\frac{13}{15} + \frac{2}{5}(0.33) + \frac{2}{5}(0.69) = 0.58$$

$$V_Y(y) = \frac{1}{5}(0.87) + \frac{2}{5}(0.33) + \frac{2}{5}(0.69) = 0.58$$

The other car options are similarly computed. From this, and the results in Table 3.1, Car 5 is the "best" choice, followed by Car 2, Car 1, Car 3, and Car 4.

Sensitivity Analysis

A common "post-analysis" of an initial ranking of alternatives is a sensitivity analysis. Often, this analysis is designed around the sensitivity of rankings to changes in the importance weights of the evaluation criteria, which in Case Discussion 3.1 are *Car Color*, *MPG*, and *Price*.

Recall that the additive value function is a weighted average of the individual single dimensional value functions of each of the evaluation criteria. Here, the weights are non-negative and sum to one. Because of this, as one weight varies, the other weights must also change such that their sum remains equal to one. An algebraic rule for automatically tracking the change in the other weights as one weight varies is described in Kirkwood [2] and is outlined below.

Consider the case of a three-term additive value model, given by Equation 3.11.

$$V_Y(y) = w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + w_3 V_{X_3}(x_3) \tag{3.11}$$

The weights in Equation 3.11 are non-negative and have the property of summing to one. One procedure for varying the weights, for the purpose of a sensitivity analysis, is the *ratio method* [2].

The ratio method operates as follows. Suppose w_2 is selected as the weight to vary. Let $w_{0,1}$ and $w_{0,3}$ denote the original set of weights for w_1 and w_3 established for the initial ranking. Then, formulas for w_1 and w_3 as a function of w_2 are, respectively

$$w_1 = (1 - w_2)\left(\frac{w_{0,1}}{w_{0,1} + w_{0,3}}\right) \quad 0 \le w_2 \le 1 \tag{3.12}$$

$$w_3 = (1 - w_2)\left(\frac{w_{0,3}}{w_{0,1} + w_{0,3}}\right) \quad 0 \le w_2 \le 1 \tag{3.13}$$

So, w_1 and w_3 will automatically change as w_2 varies. This change will be such that $w_1 + w_2 + w_3 = 1$. This formulation also keeps the values for w_1 and w_3 in the same ratio as the ratio of their original weight values; that is, it can be shown that

$$\frac{w_3}{w_1} = \frac{w_{0,3}}{w_{0,1}}.$$

In Case Discussion 3.1, recall that $w_1 = \frac{1}{5}$, $w_2 = \frac{2}{5}$, and $w_3 = \frac{2}{5}$. These were the original weights established for the initial ranking. Observe the ratio of w_3 to w_1

equals 2. This is the ratio preserved by Equations 3.12 and 3.13, where, for the sensitivity analysis, we set $w_{0,1} = \frac{1}{5}$ and $w_{0,3} = \frac{2}{5}$. Thus, for a sensitivity analysis on the weights in Case Discussion 3.1 we have the following:

$$w_1 = (1 - w_2)\left(\frac{1}{3}\right) \quad 0 \leq w_2 \leq 1 \tag{3.14}$$

$$w_3 = (1 - w_2)\left(\frac{2}{3}\right) \quad 0 \leq w_2 \leq 1 \tag{3.15}$$

In Case Discussion 3.1, suppose $w_2 = 0.2$ instead of its original value of 0.4 (or 2/5). From Equations 3.14 and 3.15 it follows that

$$w_1 = (1 - 0.2)\left(\frac{1}{3}\right) = \frac{4}{15}$$

$$w_3 = (1 - 0.2)\left(\frac{2}{3}\right) = \frac{8}{15}$$

Equation 3.10 then becomes

$$V_Y(y) = \frac{4}{15}V_{X_1}(x_1) + \frac{2}{10}V_{X_2}(x_2) + \frac{8}{15}V_{X_3}(x_3)$$

From Table 3.1, for Car 1 we have

$$V_Y(y) = \frac{4}{15}(0.87) + \frac{2}{10}(0.33) + \frac{8}{15}(0.69) = 0.67$$

Similar calculations can be done for the rest of the cars in the set of alternatives. Table 3.2 summarizes the results of these calculations, as the weight for MPG denoted by w_2 varies from zero to one in increments of 0.1.

A graph of the results in Table 3.2 is shown in Figure 3.9. Notice Car 5 is the clear winner and dominates the overall value score. Car 4 dominates the "loss column" falling below all other car option scores.

Measurement Scales

Thus far, we have said very little about the nature of the scales used for the value functions presented in the preceding discussions. This section presents a general

TABLE 3.2: Case Discussion 3.1: Sensitivity Analysis on Miles per Gallon

Sensitivity Analysis MPG Weight Variation		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	Original Case											
Car 1	0.58	0.75	0.71	0.67	0.63	0.58	0.54	0.50	0.46	0.42	0.37	0.33
Car 2	0.62	0.57	0.59	0.60	0.61	0.62	0.63	0.65	0.66	0.67	0.68	0.69
Car 3	0.55	0.58	0.57	0.56	0.55	0.55	0.54	0.53	0.52	0.52	0.51	0.50
Car 4	0.26	0.34	0.32	0.30	0.28	0.26	0.24	0.22	0.20	0.18	0.16	0.14
Car 5	0.84	0.78	0.80	0.81	0.83	0.84	0.86	0.87	0.89	0.90	0.92	0.93

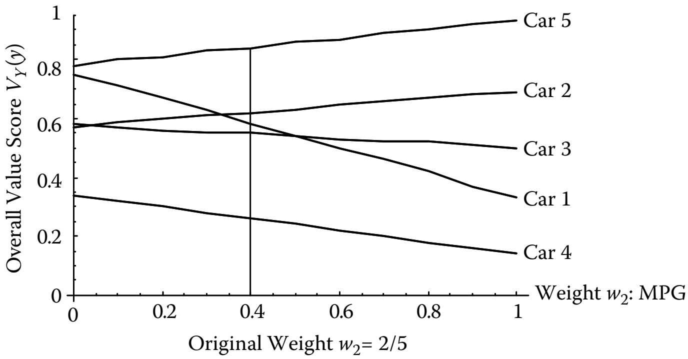


Figure 3.9: Case Discussion 3.1: sensitivity analysis on miles per gallon.

discussion on the key types of measurement scales and those that are commonly used to develop single dimensional value functions.

A *measurement scale* is a particular way of assigning numbers or labels to an attribute or measure. In measurement and decision theory there are four commonly used measurement scales [5]. These are *nominal scale*, *ordinal scale*, *interval scale*, and *ratio scale*. Each scale is described below.

Nominal Scale

A nominal scale is a measurement scale in which attributes are assigned a label (i.e., a name). It is only a qualitative scale. Nominal data can be counted but no quantitative differences or preference ordering of the attributes are implied in a nominal scale. From this, it follows that arithmetic operations are without meaning in a nominal scale. Figure 3.10 illustrates a nominal scale for a set of U.S. cities labeled A, B, C, and D.

Ordinal Scale

An ordinal scale is a measurement scale in which attributes are assigned a number that represents order or rank. For example, a person might rate the

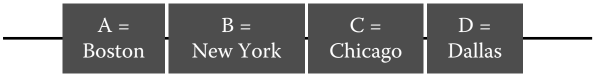


Figure 3.10: A nominal scale.

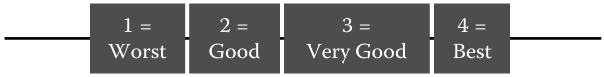


Figure 3.11: An ordinal scale.

quality of different ice cream flavors, at the local parlor, according to the scale in Figure 3.11.

Here, a scale of one to four is assigned to "Worst," "Good," "Very Good" and "Best," respectively. The numerical values indicate only relative order in the sequence. The distance between the numbers is arbitrary and has no meaning. One could have easily assigned the number 40 to the attribute Best, instead of 4, while still preserving the original order of the sequence.

In an ordinal scale, such as the one in Figure 3.11, it does not follow that Good is twice as valuable as Worst, or Best is twice as valuable as Good. We can only say that Best is more valued than Very Good, Very Good is more valued than Good, and Good is more valued than Worst but, in each case, we can't say by how much they are more valued.

Data along an ordinal scale is more insightful than data along a nominal scale because the ordinal scale provides information on preference order or rank. However, because the distance between values in an ordinal scale is arbitrary, arithmetic operations on ordinal data are impermissible.

Interval Scale

An interval scale is a measurement scale in which attributes are assigned numbers such that differences between them have meaning. The zero point on an interval scale is chosen for convenience and does not necessarily represent the absence of the attribute being measured. Examples of interval scales are the Fahrenheit or Celsius temperature scales. The zero point on these temperature scales does not mean the absence of temperature. In particular, zero degrees Celsius is assigned as the freezing point of water.

Because distances between numbers in an interval scale have meaning, addition and subtraction of interval scale numbers is permitted; however, because the zero point is arbitrary, multiplication and division of interval scale numbers is not permitted. For example, we can say that 75 degrees Fahrenheit is 25 Fahrenheit degrees hotter than 50 degrees Fahrenheit; but, we cannot say 75 degrees Fahrenheit is 50% hotter than 50 degrees Fahrenheit. *However, in an interval scale ratios of differences can be expressed meaningfully; for example, one difference can be one-half or twice or three times another difference*.

When working with *measurable value functions*, introduced in the beginning of this section, such differences are referred to as *preference differences*. Mentioned earlier, a measurable value function is one that is monotonic in preferences and value differences represent relative strength of preference. Thus, large value differences between options (alternatives) indicate the difference in preference between them is greater, say, than the difference in preference between other options (alternatives). Furthermore, the numerical amount of this difference represents the relative amount of preference difference. The concept of value differences is considered a "primitive concept" in decision theory; that is, it is a concept not derived from other conditions.*

Ratio Scale

A ratio scale is an interval measurement scale with a "true zero." Here, attributes are assigned numbers such that (1) differences between the numbers on this scale reflect differences of the attribute and (2) ratios between the numbers on this scale reflect ratios of the attribute. On a ratio scale the zero point is a "true zero." It represents a complete absence of the characteristic being measured by the attribute. All arithmetic operations are permitted on numbers that fall along a ratio scale. Examples of ratio scales include such measures as distance, weight, money. Probability is a number that falls along the ratio scale from zero to one. Also, the weights in the additive value function are defined along a ratio scale.

Cardinal Scale

A cardinal scale is a measurement scale that is either interval scaled or ratio scaled.

Natural Scale

A natural scale is a measurement scale in general use and has a widely accepted interpretation. The engineering development cost of a new system is an example

*This discussion derives from exchanges between C. W. Kirkwood [2] and the author.

of a natural scale for many engineering management decisions. The timeframe, measured in weeks, when a risk event can impact a project is an example of a natural scale. In many situations, a single dimensional value function can be represented by one of the exponential forms (see Equations 3.3, 3.4) when a natural scale appropriately describes a criterion.

Constructed Scale

A constructed scale is a measurement scale specific to the evaluation criterion being measured. Constructed scales are developed for a specific decision context. They are often defined when natural scales are not possible or are not practical to use. They are also used when natural scales exist but additional context is desired and hence are used to supplement natural scales with additional information for the decision-maker.

An example of a constructed scale was the scale for the evaluation criterion *Car Color*, discussed in the preceding sections. Other examples of constructed scales are shown in Tables 3.3 and 3.4, respectively. Table 3.3 is a constructed scale for a risk event's impact on the technical performance of a system. Table 3.4 is from Kirkwood [2]. It shows a constructed scale for the security impacts of a networking strategy for a collection of personal computers. It also shows a value function mapping from an ordinal scale to a cardinal interval scale.

The use of constructed scales is common in decision theory. Excellent examples of constructed scales can be found in Kirkwood [2], Keeney [6], and Clemen [7].

Direct and Proxy Scales

Direct Scale

A natural scale or a constructed scale can be characterized as a direct scale or a proxy scale. A *direct scale* directly measures the degree of attainment of an evaluation criterion [2]. An example of a direct scale that is also a natural scale is MPG for the evaluation criterion Fuel Efficiency.

Another way to view the constructed scale in Table 3.4 is in the form of a single dimensional value function. Figure 3.12 presents this view.

TABLE 3.3: A Constructed Scale for a Risk Event's Technical Performance Impact

Ordinal Scale Level (Score)	Definition/Context: Technical Performance Impact
5	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that critical technical performance (or system capability) shortfalls result.
4	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that technical performance (or system capability) is marginally below minimum acceptable levels.
3	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering or these capabilities) to the extent that technical performance (or system capability) falls well below stated objectives but remains enough above minimum acceptable levels.
2	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that technical performance (or system capability) falls below stated objectives but well above minimum acceptable levels.
1	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) in a way that results in a negligible effect on overall performance (or achieving capability objectives for a build/block/increment), but regular monitoring for change is strongly recommended.

Proxy Scale

As the name implies, a *proxy scale* indirectly measures the degree of attainment of an evaluation criterion; it does not directly measure it. A common example of a proxy scale is gross national product (GNP) as a way to measure the economic health of a nation.

TABLE 3.4: A Constructed Scale for Security Impact [2]

Ordinal Scale Level (Score)	**Definition/Context Network Strategy**	**Value Function (Cardinal Interval Scale)**
-2	The addition of the network causes a potentially serious decrease in system control and security for the use of data or software.	$V_X(-2) = 0$
-1	There is a noticeable but acceptable diminishing of system control and security.	$V_X(-1) = 0.50$
0	There is no detectable change in system control or security.	$V_X(0) = 0.83$
1	System control or security is enhanced by the addition of a network.	$V_X(1) = 1$

3.3 Risk and Utility Functions

In the preceding section, uncertainty was not considered in the analysis that led to the identification of the "best" alternative from a set of competing alternatives. The specification and application of value functions is appropriate when there is *certainty* in how an alternative (or option) rates across the levels (or scores) for the criteria that define the decision problem. What if there is *uncertainty* in how these alternatives perform across these levels? How should decisions be made in the presence of uncertainty? This is the main theme of this section. Here, we discuss

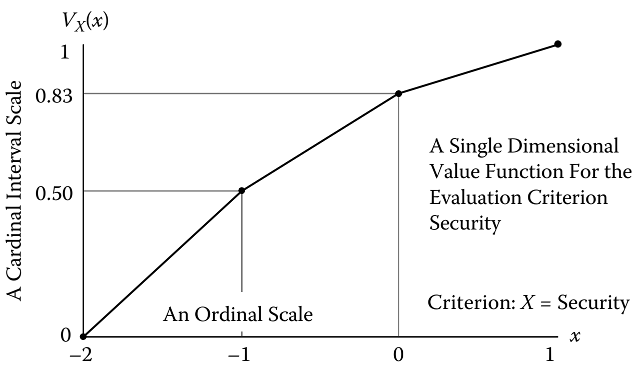


Figure 3.12: A single dimensional value function for security impact [2].

how to incorporate uncertainty into the decision problem. Among the topics and concepts introduced will be risk, risk attitudes, and utility functions.

Risk

Risk is an event that, if it occurs, has unwanted consequences. Risk is a probabilistic event. A risk event may or may not occur with some probability p.

Risk can be studied in terms of a person's attitude in taking chances or gambling on outcomes in business or in any type of decision situation.

Lotteries and Risk Attitudes

Risk can be characterized in the way a person evaluates uncertain outcomes. Uncertain outcomes can be portrayed as a lottery. A *lottery* is an event whose outcome is determined by chance. A lottery is sometimes called a *gamble* or *risky prospect*. Formally, a lottery can be written as

$$\textit{Lottery}\; X = \begin{cases} x_1 & \text{with probability } p_1 \\ x_2 & \text{with probability } p_2 \\ x_3 & \text{with probability } p_3 \end{cases}$$

where X is an event whose outcome (or consequence) is x_1 with probability p_1, x_2 with probability p_2, or x_3 with probability p_3. Here, the sum of these probabilities is one; that is, $p_1 + p_2 + p_3 = 1$.

People evaluate lotteries in a number of ways. One way is to compute the expected value (or expected outcome) of a lottery. This leads to the following definition.

Definition 3.7 The *expected value* $E(X)$ of lottery X with possible outcomes $\{x_1, x_2, x_3, \ldots, x_n\}$ is

$$E(X) = p_1 x_1 + p_2 x_2 + p_3 x_3 + \cdots + p_n x_n \tag{3.16}$$

where p_i is the probability X takes the value x_i, for $i = 1, \ldots, n$.

The decision to participate in a lottery rests with a person's willingness (or lack of) to take risks. One way to characterize a person's risk attitude is through the concept of certainty equivalent, which is defined next.

Definition 3.8 A *certainty equivalent* of lottery X is an amount x_{CE} such that the decision-maker is indifferent between X and the amount x_{CE} for certain [1].

For example, given the lottery below what amount of money would you be willing to receive with certainty that makes you indifferent between that amount and engaging in lottery X?

$$\textit{Lottery } X = \begin{cases} \text{Win \$500 with probability 0.6} \\ \text{Lose \$150 with probability 0.4} \end{cases}$$

Here, the expected value of this lottery is

$$E(X) = 0.6(\$500) + 0.4(-\$150) = \$240$$

If you would be indifferent between receiving 200 dollars with certainty and engaging in the lottery, then we say your certainty equivalent for this lottery is $x_{\text{CE}} = 200$.

In this case, we say this person is *risk averse*. He or she is willing to accept, with certainty, an amount of money less than the expected amount that might be received if the decision were made to participate in the lottery (or gamble). People are considered to be risk takers or *risk seeking* if their certainty equivalent is greater than the expected value of the lottery. People are considered *risk neutral* if their certainty equivalent is equal to the expected value of the lottery.

There is a mathematical relationship among certainty equivalent, expected value, and risk attitude. People with increasing preferences whose risk attitude is *risk averse* will always have a certainty equivalent less than the expected value of an outcome. People with decreasing preferences whose risk attitude is *risk averse* will always have a certainty equivalent greater than the expected value of an outcome.

People with increasing preferences whose risk attitude is *risk seeking* will always have a certainty equivalent greater than the expected value of an outcome. People with decreasing preferences whose risk attitude is *risk seeking* will always have a certainty equivalent less than the expected value of an outcome.

People whose risk attitude is *risk neutral* will always have a certainty equivalent equal to the expected value of an outcome. This is true regardless of whether a person has increasing or decreasing preferences.

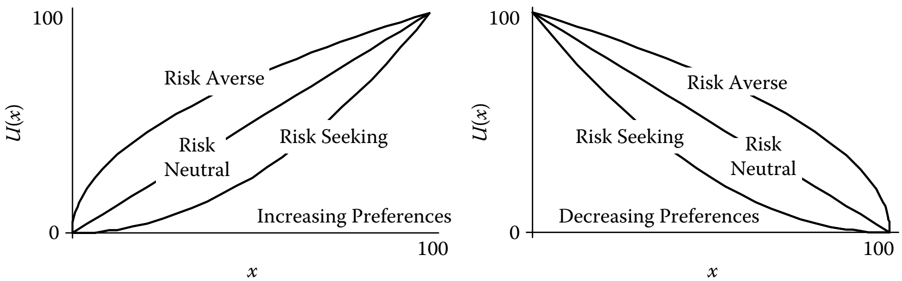


Figure 3.13: Families of risk attitude or utility functions.

A class of mathematical functions exist that exhibit behaviors with respect to risk-averse, risk-seeking, and risk-neutral attitudes. These are referred to as *utility functions*. A family of such functions is shown in Figure 3.13.

Utility and Utility Functions

A *utility* is a measure of worth, satisfaction, or preference an outcome has for an individual. It is a dimensionless number that is sometimes referred to as a "util." A *utility function* is a real-valued mathematical function that relates outcomes along the horizontal axis to measures of worth or utils along the vertical axis.*

The vertical axis of a utility function can range across the real number line; however, this axis is usually scaled to run between 0 and 100 or 0 to 1. With this convention, the utility of the least preferred outcome (or option) is assigned the number zero and the utility of the most preferred outcome (or option) is assigned the number one. Higher preferred outcomes have higher utils than lower preferred outcomes.

Utility functions generally take one of the shapes shown in Figure 3.13. They are concave, linear, or convex. A concave utility function appears "hill-like" and is always associated with a risk-averse person. Concave functions lie above a chord drawn between any two points on the curve. A linear function is always associated with a risk-neutral person. A convex function appears "bowl-like" and is always associated with a risk-seeking person. Convex functions lie below a chord drawn between any two points on the curve. It is important to note that the certainty equivalent of any lottery is unique for monotonic utility functions [1].

*A utility function is a value function but a value function is not necessarily a utility function [1].

Expected Utility and Certainty Equivalent

Utility functions, as representations of a person's risk attitude, exhibit a number of relationships between the utility of the expected value of a lottery and the expected utility of a lottery. Both measures can be related to the concept of certainty equivalent. Next, we introduce the concept of expected utility.

Definition 3.9 The *expected utility* of lottery X with utilities $U(x_1)$, $U(x_2)$, $U(x_3), \ldots, U(x_n)$ of possible outcomes $\{x_1, x_2, x_3, \ldots, x_n\}$ is

$$E(U(x)) = p_1U(x_1) + p_2U(x_2) + p_3U(x_3) + \cdots + p_nU(x_n) \tag{3.17}$$

The relationship between the expected value $E(X)$ of a lottery and the expected utility $E(U(x))$ of a lottery can be seen by looking at the utility function in Figure 3.14. Figure 3.14 shows this relationship for a monotonically increasing risk averse utility function. Similar relationships can be developed for other utility function shapes, such as those in Figure 3.13.

In Figure 3.14, the equation of the chord is given by Equation 3.18.

$$y_{\text{Chord}}(x) = m(x-b) + U(b) \quad \text{where} \quad m = \frac{U(b)-U(a)}{b-a} \tag{3.18}$$

Here, we have $E(X) = pa + (1-p)b$. If we set $x = E(X)$ in the equation of the chord then, with a little algebra, it can be shown that

$$y_{\text{Chord}}(E(X)) = pU(a) + (1-p)U(b) = E(U(x)) \tag{3.19}$$

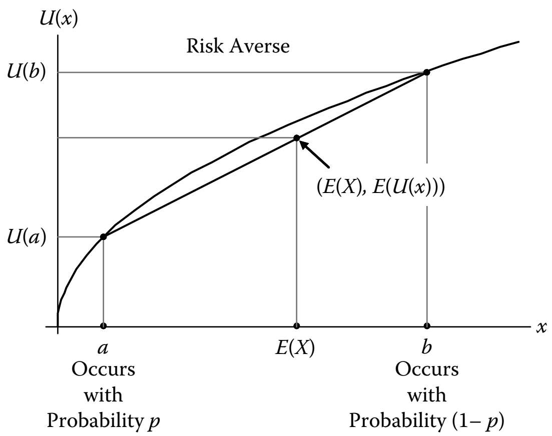


Figure 3.14: Relationship between $E(X)$ and $E(U(x))$.

Recall that the certainty equivalent x_{CE} is that value on the horizontal axis where a person is indifferent between a lottery and receiving the amount x_{CE} with certainty. From this, it follows that the utility of x_{CE} must equal the expected utility of a lottery; that is,

$$U(\mathrm{x}_{CE}) = E(U(x)) \tag{3.20}$$

or, equivalently

$$y_{\text{Chord}}(E(X)) = U(\mathrm{x}_{CE}) \tag{3.21}$$

as shown in Figure 3.15.

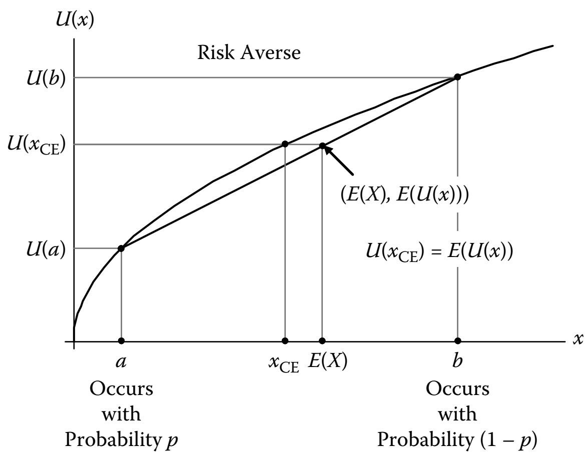


Figure 3.15: Relationship between $E(U(x))$, $E(X)$, and x_{CE}.

From Equation 3.20, it follows that when a utility function has been specified, the certainty equivalent x_{CE} can be solved for by taking the utility function's inverse; that is,

$$x_{CE} = U^{-1}(E(U(x))) \tag{3.22}$$

where U^{-1} is the inverse of the utility function.

Example 3.1 Consider the lottery X given below.

$$\textit{Lottery } X = \begin{cases} \text{Win \$80K with probability 0.6} \\ \text{Win \$10K with probability 0.4} \end{cases}$$

Determine the certainty equivalent x_{CE} for this lottery if a person's utility function is given by $U(x) = 10\sqrt{x}$, where x is in dollars thousand (K).

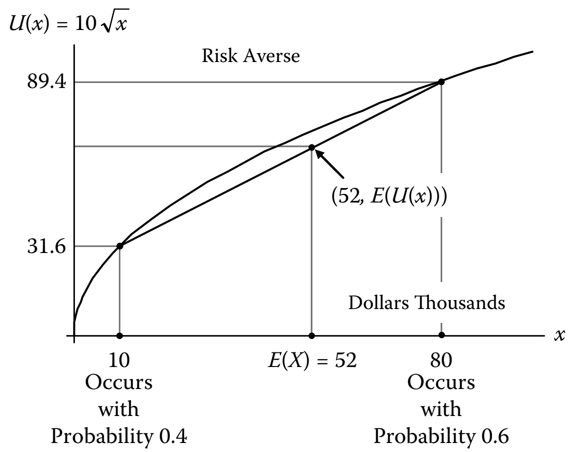


Figure 3.16: Example 3.1 x_{CE} derivation.

Solution

A graph of this function reveals preferences that are monotonically increasing and risk averse, as shown in Figure 3.16. First, we compute the expected value of the lottery; that is,

$$E(X) = p_1x_1 + p_2x_2 = 0.6(\$80\text{K}) + 0.4(\$10\text{K}) = \$52\text{K}$$

We know that x_{CE} will be less than $E(X)$ since $U(x)$ is a monotonically increasing risk averse utility function. From Equation 3.18, the equation of the chord is

$$y_{\text{Chord}}(x) = m(x - 80) + U(80) \quad \text{where} \quad m = \frac{U(80) - U(10)}{80 - 10}$$

$$y_{\text{Chord}}(x) = m(x - 80) + 89.4 \quad \text{where} \quad m = \frac{89.4 - 31.6}{80 - 10} = 0.8257$$

From Equation 3.19, we know that

$$y_{\text{Chord}}(E(X)) = pU(a) + (1 - p)U(b) = E(U(x))$$

which, in this case, is

$$y_{\text{Chord}}(52) = 0.8257(52 - 80) + 89.4 = 66.3 = E(U(x))$$

Note that if we used the definition of expected utility we would get the same result; that is,

$$E(U(x)) = p_1 U(10) + p_2 U(80) = 0.4(31.6) + 0.6(89.4) = 66.3$$

where $p_2 = (1 - p_1)$. From Equation 3.20 we know that

$$U(x_{\text{CE}}) = E(U(x)) = 66.3$$

Given $U(x) = 10\sqrt{x}$ then the inverse of this utility function is $U^{-1}(x) = (x/10)^2$. From this, it follows that

$$x_{\text{CE}} = U^{-1}(E(U(x)) = (66.3/10)^2 = 43.93 \approx 44$$

This example is summarized graphically in Figure 3.17.

In Figure 3.17 notice there is a point (a dot) just above the coordinate (52, 66.3). This point denotes the utility of the expected value; that is, $U(E(X))$. A property of concave functions (risk-averse utility functions) is

$$U(E(X)) > E(U(x))$$

Likewise, a property of convex functions (risk seeking utility functions) is

$$U(E(X)) < E(U(x))$$

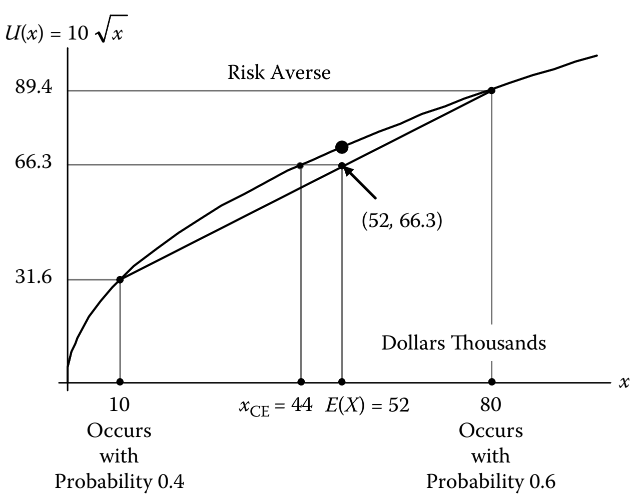


Figure 3.17: Example 3.1 summary.

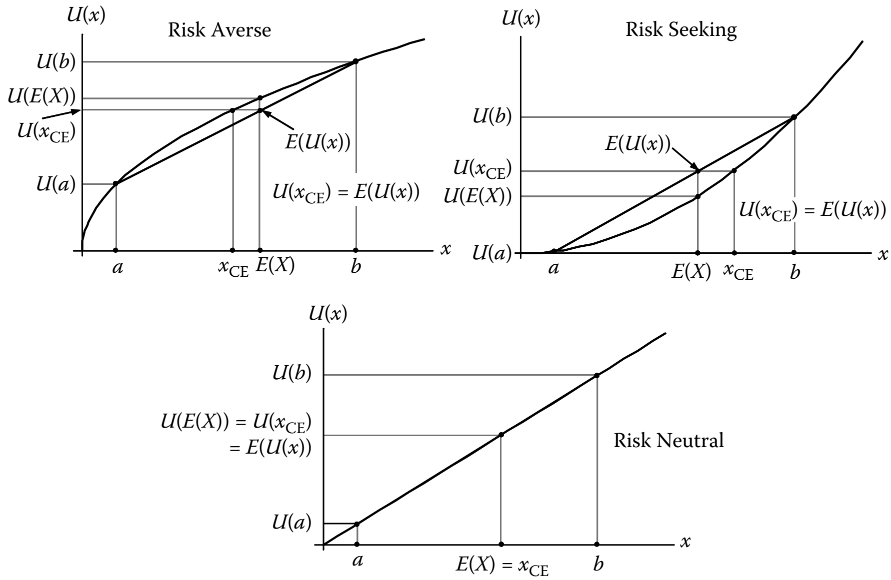


Figure 3.18: A family of utility functions for monotonically increasing preferences.

Risk-neutral utility functions have the property

$$U(E(X)) = E(U(x))$$

Figure 3.18 summarizes these relationships graphically for monotonically increasing preferences.

Finally, the following is worth noting. Suppose a utility function is scaled such that its vertical axis ranges from zero to one. Suppose a lottery has two outcomes. Outcome a occurs with probability p. Outcome b occurs with probability $(1-p)$. If preferences are monotonically increasing (i.e., more is better than less) such that $U(a) = 0$ and $U(b) = 1$, then it can be shown that

$$E(U(x)) = 1 - p \tag{3.23}$$

Similarly, if the utility function is monotonically decreasing, (i.e., less is better than more) such that $U(a) = 1$ and $U(b) = 0$, then it can be shown that

$$E(U(x)) = p \tag{3.24}$$

The Exponential Utility Function

A special type of utility function known as the exponential utility function [2] can represent a broad class of utility function shapes or risk attitudes. Similar in form to the exponential value function, the exponential utility function is given below.

Definition 3.10 If utilities are monotonically increasing over the levels (scores) for an evaluation criterion X, then the exponential utility function is given by

$$U(x) = \begin{cases} \dfrac{1 - e^{-(x - x_{\min})/\rho}}{1 - e^{-(x_{\max} - x_{\min})/\rho}} & \text{if } \rho \neq \infty \\ \dfrac{x - x_{\min}}{x_{\max} - x_{\min}} & \text{if } \rho = \infty \end{cases} \tag{3.25}$$

Definition 3.11 If utilities are monotonically decreasing over the levels (scores) for an evaluation criterion X, then the exponential utility function is given by

$$U(x) = \begin{cases} \dfrac{1 - e^{-(x_{\max} - x)/\rho}}{1 - e^{-(x_{\max} - x_{\min})/\rho}} & \text{if } \rho \neq \infty \\ \dfrac{x_{\max} - x}{x_{\max} - x_{\min}} & \text{if } \rho = \infty \end{cases} \tag{3.26}$$

The function $U(x)$ is scaled such that it ranges from zero to one. In particular, for monotonically increasing preferences $U(x_{\min}) = 0$ and $U(x_{\max}) = 1$. The opposite holds for monotonically decreasing preferences; that is, $U(x_{\min}) = 1$ and $U(x_{\max}) = 0$.

A family of exponential utility functions is shown in Figure 3.19. The left-most picture reflects exponential utility functions for monotonically increasing

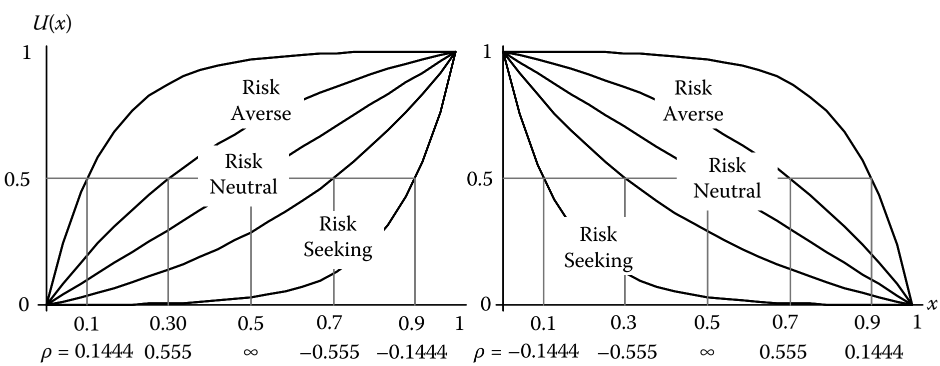


Figure 3.19: Families of exponential utility functions.

preferences ("more is better") over the criterion X. The right-most picture reflects exponential utility functions for monotonically decreasing preferences ("less is better) over the criterion X.

In Equations 3.25 and 3.26, the constant ρ is called the *risk tolerance*. The risk tolerance ρ reflects the risk attitude of a person's utility or preferences for a particular outcome. Positive values of ρ reflect a risk-averse utility function. Negative values of ρ reflect a risk-seeking utility function. A ρ-value of "infinity" reflects a risk-neutral utility function.

Working with the Exponential Utility Function

Mentioned previously, an exponential utility function can be specified to represent many shapes that reflect a person's risk attitude. The shape is governed by ρ, whose magnitude reflects the degree a person is risk averse or risk seeking. Of course, if a person is neither risk averse nor risk seeking, then the exponential utility function becomes a straight line. The following discusses this further and provides ways to determine the shape of an exponential utility function when either ρ or the certainty equivalent x_{CE} is known or given.

Determining the Risk Tolerance ρ from the Certainty Equivalent

Consider an investment with the following two outcomes. Earn 10 million dollars (\$M) with probability $p = 1/3$ or earn 20 million dollars (\$M) with probability $(1 - p) = 2/3$. Suppose $U(10) = 0$, $U(20) = 1$, and the certainty equivalent for this lottery was set at 13 million dollars. What is the value of ρ?

To answer this question, first determine the expected earnings from this investment. From Equation 3.16, the expected earnings are:

$$\text{Expected Earnings } E(X) = (1/3)(\$10\text{M}) + (2/3)(\$20\text{M}) = \$16.67\text{M}$$

Because the certainty equivalent, in this case, was decided to be set at \$13M, we know this investor is risk averse. Why? Because, in this case, $x_{CE} < E(X)$. We also have monotonically increasing preferences because earning more is better than earning less. So, the utility function should look something like one of the upper curves in the left-hand side of Figure 3.19. In this case,

$$U(x) = \frac{1 - e^{-(x-10)/\rho}}{1 - e^{-(20-10)/\rho}} = \frac{1 - e^{-(x-10)/\rho}}{1 - e^{-(10)/\rho}} \tag{3.27}$$

Next, compute the expected utility $E(U(x))$. In this case, Equation 3.23 applies; that is,

$$E(U(x)) = 1 - p = 2/3 \tag{3.28}$$

Now, we know from Equation 3.20 that

$$U(x_{\mathrm{CE}}) = E(U(x)) = 2/3 \tag{3.29}$$

Since x_{CE} was given to be equal to \$13M, from Equation 3.27 it follows that

$$U(x_{\mathrm{CE}}) = U(13) = \frac{1 - e^{-(13-10)/\rho}}{1 - e^{-(20-10)/\rho}} = \frac{1 - e^{-(3)/\rho}}{1 - e^{-(10)/\rho}} = 2/3 \tag{3.30}$$

Solving Equation 3.30 numerically for ρ yields $\rho = 2.89139$. This was done using the *Mathematica*® routine

FindRoot $[(1 - \mathrm{Exp}[-3/\rho])/(1 - \mathrm{Exp}[-10/\rho]) == 2/3, \{\rho, 1\}]$

which returns the value $\rho = 2.89139$. A graph of this exponential utility function is shown in Figure 3.20.

From this discussion we see that, when specifying an exponential utility function, it is necessary to identify the x-value associated with the certainty equivalent. Once the certainty equivalent has been specified, the shape of the exponential utility function, which reflects the risk attitude of the individual, can be completely

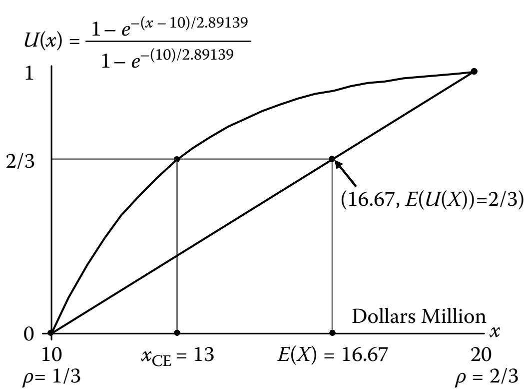


Figure 3.20: An exponential utility function.

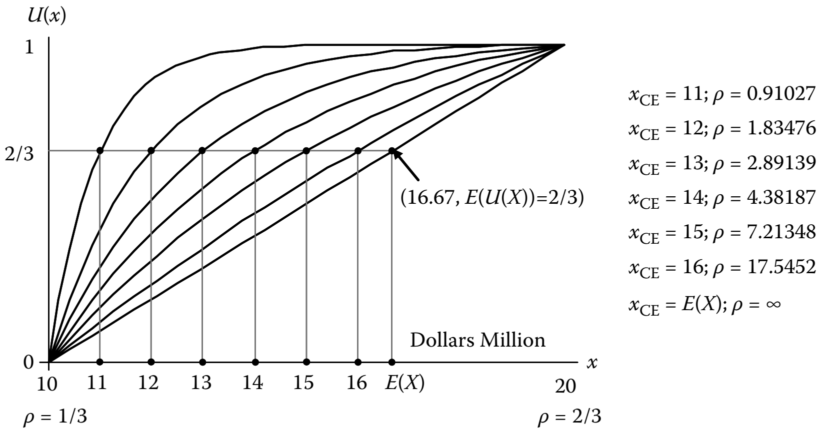


Figure 3.21: Families of risk-averse exponential utility functions.

determined. Figure 3.21 shows a family of exponential utility functions for various certainty equivalents as they vary around the basic data in Figure 3.20. Notice the increased sharpness in risk averseness as the certainty equivalent moves to the left of $E(X)$, in this case.

Determining the Certainty Equivalent from the Risk Tolerance ρ

Suppose an exponential utility function has been defined for a specific ρ. From this, we will develop a rule for computing the implied certainty equivalent x_{CE}.

Recall from Equation 3.22 that when a utility function has been specified, the certainty equivalent x_{CE} can be solved for by taking the utility function's inverse; that is,

$$x_{CE} = U^{-1}(E(U(x))) \tag{3.31}$$

where U^{-1} is the inverse of the utility function.

Suppose we started off with the utility function specified in Figure 3.20 and we wanted to know the certainty equivalent. Here, we have a value for ρ and we want the value for x_{CE} given ρ.

To determine x_{CE}, it is necessary to apply Equation 3.31 to the exponential utility function. We need two formulas for this. One is the formula for the inverse of the exponential utility function. The other is the expected utility of the exponential utility function. These formulas are provided below. It is left to the reader to derive these results, which involves only some algebraic manipulations.

Theorem 3.2 *Given a monotonically increasing exponential utility function, the inverse function, expected utility, and certainty equivalent are as follows:*

(a) Inverse Function

$$U^{-1}(x) = \begin{cases} x_{\min} - \rho \ln(1 - x/k) & \text{if } \rho \neq \infty \\ x(x_{\max} - x_{\min}) + x_{\min} & \text{if } \rho = \infty \end{cases} \tag{3.32}$$

where $k = 1/(1 - e^{-(x_{\max} - x_{\min})/\rho})$

(b) Expected Utility

$$E(U(x)) = \begin{cases} k(1 - e^{x_{\min}/\rho} E(e^{-x/\rho})) & \text{if } \rho \neq \infty \\ \dfrac{E(X) - x_{\min}}{x_{\max} - x_{\min}} & \text{if } \rho = \infty \end{cases} \tag{3.33}$$

(c) Certainty Equivalent

$$x_{\text{CE}} = \begin{cases} -\rho \ln E(e^{-x/\rho}) & \text{if } \rho \neq \infty \\ E(X) & \text{if } \rho = \infty \end{cases} \tag{3.34}$$

Theorem 3.3 *Given a monotonically decreasing exponential utility function then the inverse function, expected utility, and certainty equivalent are as follows:*

(a) Inverse Function

$$U^{-1}(x) = \begin{cases} x_{\max} + \rho \ln(1 - x/k) & \text{if } \rho \neq \infty \\ x_{\max} - x(x_{\max} - x_{\min}) & \text{if } \rho = \infty \end{cases} \tag{3.35}$$

where $k = 1/(1 - e^{-(x_{\max} - x_{\min})/\rho})$

(b) Expected Utility

$$E(U(x)) = \begin{cases} k(1 - e^{-x_{\max}/\rho} E(e^{x/\rho})) & \text{if } \rho \neq \infty \\ \dfrac{x_{\max} - E(X)}{x_{\max} - x_{\min}} & \text{if } \rho = \infty \end{cases} \tag{3.36}$$

(c) Certainty Equivalent

$$x_{\text{CE}} = \begin{cases} \rho \ln E(e^{x/\rho}) & \text{if } \rho \neq \infty \\ E(X) & \text{if } \rho = \infty \end{cases} \tag{3.37}$$

Example 3.2 Consider the utility function in Figure 3.20. Show that the certainty equivalent for this utility function is \$13M.

Solution The utility function in Figure 3.20 is given by

$$U(x) = \frac{1 - e^{-(x-10)/2.89139}}{1 - e^{-(10)/2.89139}} = 1.0325(1 - e^{-(x-10)/2.89139}) \tag{3.38}$$

where $\rho = 2.89139$. Since this function is monotonically increasing, its certainty equivalent x_{CE} is given by Equation 3.34. Applying that equation, with reference to Figure 3.20, we have

$$\mathrm{x}_{CE} = -\rho \ln E(e^{-x/\rho}) = -2.89139 \ln((1/3)e^{-10/2.89139} + (2/3)e^{-20/2.89139}) = 13$$

So, the certainty equivalent of the exponential value function is \$13M, as shown in Figure 3.20.

Thus far, we have worked with lotteries that represent uncertain events having a discrete number of chance outcomes. When the outcomes of a lottery are defined by a continuous probability density function, then Equations 3.16 and 3.17 become the following:

$$E(X) = \int_a^b x f_X(x)\, dx \tag{3.39}$$

$$E(U(x)) = \int_a^b U(x) f_X(x)\, dx \tag{3.40}$$

Furthermore, the certainty equivalent x_{CE} becomes the solution to

$$U(x_{CE}) = E(U(x)) = \int_a^b U(x) f_X(x)\, dx \tag{3.41}$$

Example 3.3 Consider the utility function in Figure 3.16, given by $U(x)$ below,

$$U(x) = 10\sqrt{x}$$

where x is in dollars thousand (K). Determine $E(X)$ and the certainty equivalent x_{CE} if lottery X is described by the uniform probability density function in Figure 3.22.

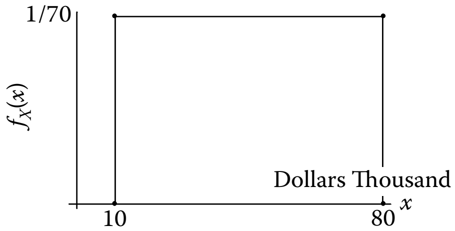


Figure 3.22: A uniform probability density function.

Solution The equation for $f_X(x)$ in Figure 3.22 is

$$f_X(x) = \frac{1}{70} \quad 10 \leq x \leq 80$$

From Equation 3.39 we have

$$E(X) = \int_a^b x f_X(x)\,dx = \int_{10}^{80} x\,(1/70)\,dx = 45$$

so, the expected value of the lottery X is \$45K. From Equation 3.40 we have

$$E(U(x)) = \int_a^b U(x) f_X(x)\,dx = \int_{10}^{80} 10\sqrt{x}(1/70)\,dx = 65.135$$

From Equation 3.20 (and Equation 3.41) we have

$$U(x_{\mathrm{CE}}) = E(U(x)) = 65.135$$

Since $U(x) = 10\sqrt{x}$ it follows that $U(x_{\mathrm{CE}}) = 10\sqrt{x_{\mathrm{CE}}} = 65.135$. Solving this equation for x_{CE} yields $x_{\mathrm{CE}} = 42.425$; that is, the certainty equivalent is \$42.43K when rounded. Notice these results are consistent with those derived for Example 3.1.

Direct Specification of Utility

A technique known as the 5-point method is sometimes used to subjectively specify a set of utility points, from which a utility function can then be inferred or drawn through these points. The approach works as follows:

Suppose you have the lottery X, given below.

$$Lottery\ X = \begin{cases} \text{Win \$500 with probability 0.5} \\ \text{Lose \$150 with probability 0.5} \end{cases}$$

Step 1. Set $U(-150) = 0$ and $U(500) = 1$. Determine the decision-maker's certainty equivalent for this lottery. Suppose x_{CE} was assessed at 100 dollars. From Equation 3.20 we know that

$$U(\text{x}_{\text{CE}}) = E(U(x))$$

For lottery X this is

$$U(\text{x}_{\text{CE}}) = E(U(x)) = \frac{1}{2}U(-150) + \frac{1}{2}U(500) = \frac{1}{2}(0) + \frac{1}{2}(1) = \frac{1}{2}$$

Since x_{CE} was assessed at 100 dollars we have, in this case,

$$U(x_{\text{CE}} = 100) = U(100) = \frac{1}{2}$$

Step 2. Next, with $U(100) = 1/2$ and $U(500) = 1$ form the new lottery

$$Lottery\ X_{0.75} = \begin{cases} \text{Win \$500 with probability 0.5} \\ \text{Win \$100 with probability 0.5} \end{cases}$$

Determine the decision-maker's certainty equivalent for this lottery. Suppose x_{CE} was assessed at 200 dollars. From Equation 3.20 we know that

$$U(\text{x}_{\text{CE}}) = E(U(x))$$

For lottery $X_{0.75}$ this is

$$U(\text{x}_{\text{CE}}) = E(U(x)) = \frac{1}{2}U(100) + \frac{1}{2}U(500) = \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}(1) = \frac{3}{4}$$

Since x_{CE} was assessed at 200 dollars we have, in this case,

$$U(x_{\text{CE}} = 200) = U(200) = \frac{3}{4}$$

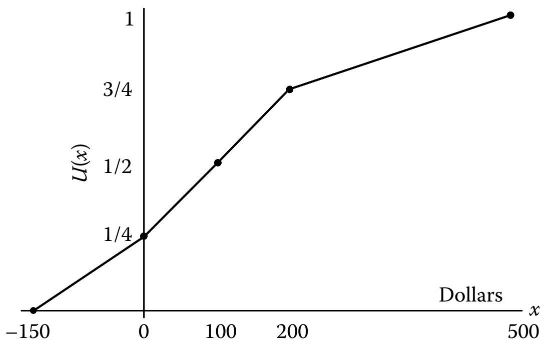


Figure 3.23: A 5-point defined utility function.

Step 3. Last, with $U(-150) = 0$ and $U(100) = 1/2$ form the new lottery

$$\textit{Lottery}\ X_{0.25} = \begin{cases} \text{Win \$100 with probability 0.5} \\ \text{Lose \$150 with probability 0.5} \end{cases}$$

Determine the decision-maker's certainty equivalent for this lottery. Suppose x_{CE} was assessed at zero dollars. From Equation 3.20, we know that

$$U(\mathrm{x}_{CE}) = E(U(x))$$

For lottery $X_{0.25}$ this is

$$U(\mathrm{x}_{CE}) = E(U(x)) = \frac{1}{2}U(-150) + \frac{1}{2}U(100) = \frac{1}{2}(0) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

Since x_{CE} was assessed at zero dollars we have, in this case,

$$U(x_{CE} = 0) = U(0) = \frac{1}{4}$$

A graph of this utility function is shown in Figure 3.23.

Multiattribute Utility and the Power-Additive Utility Function

Thus far, we have looked at utility from a single dimensional perspective. The preceding discussion and problems were focused on a single attribute utility function, its properties, and its characteristics. This section looks at multiattribute utility. Multiattribute utility is concerned with specifying a utility function over multiple attributes or multiple evaluation criteria that characterize an option or alternative.

The question is one of ranking these options or alternatives as a function of how well they perform across a set of attributes or evaluation criteria.

Multiattribute utility functions come in several general forms [6]. For purposes of this discussion, we will focus on one specific form known as the *power-additive utility function*. Kirkwood [2] provides an extensive discussion on this utility function, from which some of this material derives. The reader is directed to references 1, 6, 7, and 8 for a discussion on the other general forms of the multiattribute utility function and the circumstances under which these forms apply.

When accounting for the risk attitude of a decision-maker it is necessary to convert "values" from a value function into utilities. Doing this requires a function that takes values from a multiattribute value function and maps them into a corresponding set of utilities. The power-additive utility function is a multiattribute utility function that performs this mapping. The power-additive utility function covers a wide span of possible risk attitudes. It is mathematically equivalent to the classical multiattribute utility functions [2]. As we shall see, working with the power-additive utility function is relatively easy and, in practice, is well-suited to ranking options or alternatives across multiple criteria in the presence of uncertainty.

The Power-Additive Utility Function

The power-additive utility function is a multiattribute utility function similar in form to the exponential value function and the exponential utility function, which have been previously discussed. The power-additive utility function has been written extensively in Kirkwood [2] where it is argued that in many practical decision-making situations it is appropriate to use an exponential form for a utility function. Furthermore, the parameters of the exponential utility function can be adjusted to reflect the span of risk attitudes that characterize a decision-maker. The following defines the power-additive utility function for monotonically increasing and decreasing preferences.

Definition 3.12 If utilities are *monotonically increasing* over the values of the additive value function $V_Y(y)$ then the power-additive utility function is given by

$$U(v) = \begin{cases} K(1 - e^{-(V_Y(y)/\rho_m)}) & \text{if } \rho_m \neq \infty \\ V_Y(y) & \text{if } \rho_m = \infty \end{cases} \tag{3.42}$$

where $K = 1/(1 - e^{-1/\rho_m})$ and $v = V_Y(y) = \sum_{i=1}^{n} w_i V_{X_i}(x_i)$ where $V_Y(y)$ is the additive value function given in Definition 3.6.

Definition 3.13 If utilities are *monotonically decreasing* over the values of the additive value function $V_Y(y)$ then the power-additive utility function is given by

$$U(v) = \begin{cases} K(1 - e^{-((1-V_Y(y))/\rho_m)}) & \text{if } \rho_m \neq \infty \\ 1 - V_Y(y) & \text{if } \rho_m = \infty \end{cases} \tag{3.43}$$

where $K = 1/(1 - e^{-1/\rho_m})$ and $v = V_Y(y) = \sum_{i=1}^{n} w_i V_{X_i}(x_i)$ where $V_Y(y)$ is the additive value function given in Definition 3.6.

Mentioned above, the value function $V_Y(y)$ is an additive value function; that is, there exists n-single dimensional value functions $V_{X_1}(x_1)$, $V_{X_2}(x_2)$, $V_{X_3}(x_3), \ldots,$ $V_{X_n}(x_n)$ satisfying

$$V_Y(y) = w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + w_3 V_{X_3}(x_3) + \cdots + w_n V_{X_n}(x_n)$$

where w_i for $i = 1, \ldots, n$ are non-negative weights (importance weights) whose values range between zero and one and where

$$w_1 + w_2 + w_3 + \cdots + w_n = 1$$

Given the conventions that (1) the single dimensional value functions $V_{X_1}(x_1)$, $V_{X_2}(x_2)$, $V_{X_3}(x_3)$, ... , $V_{X_n}(x_n)$ each range in value between zero and one and (2) the weights each range in value between zero and one and sum to unity it follows that $V_Y(y)$ will range between zero and one. From this, it also follows that the power-additive utility function will also range between zero and one.

In the above definitions we assume that conditions for an additive value function hold, as well as an independence condition known as *utility independence*. *Utility independence* is a stronger form of independence than preferential independence. From Clemen [7] an attribute X_1 is *utility independent* of attribute X_2 if preferences for uncertain choices involving different levels of X_1 are independent of the value of X_2.

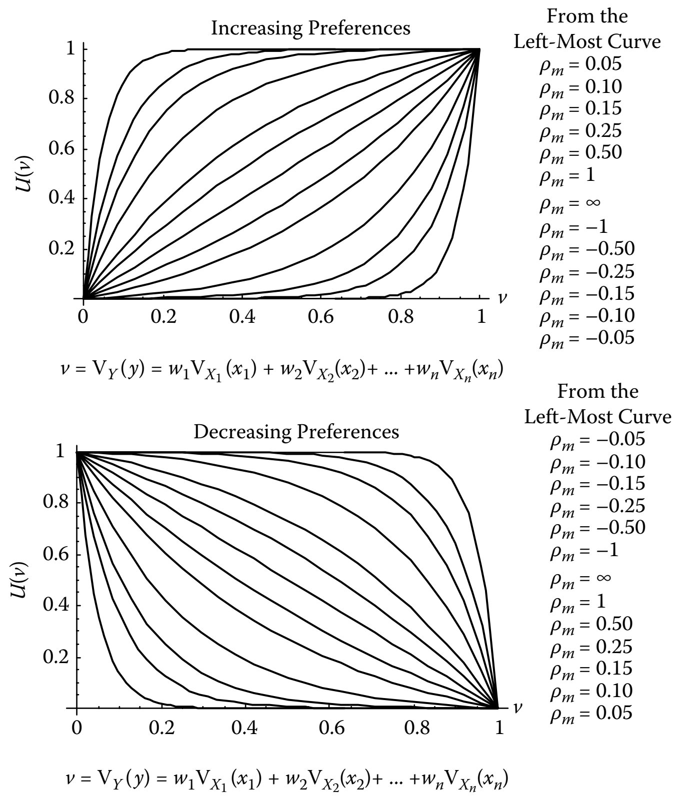


Figure 3.24: Families of power-additive utility functions.

Working with the Power-Additive Utility Function

The shape of the power-additive utility function is governed by a parameter known as the *multiattribute risk tolerance* ρ_m [2]. Figure 3.24 presents families of power-additive utility functions for various ρ_m and for increasing or decreasing preferences. A multiattribute risk-averse utility function has a positive value for ρ_m. A multiattribute risk-seeking utility function has a negative value for ρ_m. The multiattribute risk-neutral case occurs when ρ_m approaches infinity. Here, we have a straight line; this is where the expected value of the value function $V_Y(y)$ can be used to rank alternatives.

One approach to selecting ρ_m is to have the decision-maker review Figure 3.24 and select the value that most reflects his/her risk attitude. An extremely risk averse decision-maker, where monotonically increasing preferences apply, might select ρ_m in the interval $0.05 \leq \rho_m \leq 0.15$. A less risk averse decision-maker, where monotonically increasing preferences apply, might select ρ_m in the interval $0.15 < \rho_m \leq 1$. As ρ_m becomes increasingly large (i.e., approaches infinity) the decision-maker is increasingly risk neutral and the power-additive utility function approaches a straight line. As we shall see, when this occurs the expected value of the value function $V_Y(y)$ can be used to rank alternatives.

The above discussion is essentially a "look-up" procedure for selecting ρ_m. Alternative approaches involve the use of lotteries similar to those previously discussed. For a discussion on the use of lotteries to derive ρ_m see Kirkwood [2].

Theorem 3.4 *If utilities are monotonically increasing over the values of the additive value function $V_Y(y)$ with the power-additive utility function given below*

$$U(v) = \begin{cases} K(1 - e^{-(V_Y(y)/\rho_m)}) & \text{if } \rho_m \neq \infty \\ V_Y(y) & \text{if } \rho_m = \infty \end{cases}$$

where $K = 1/(1 - e^{-1/\rho_m})$ and $v = V_Y(y) = \sum_{i=1}^{n} w_i V_{X_i}(x_i)$ then

$$E(U(v)) = \begin{cases} K(1 - E(e^{-(V_Y(y)/\rho_m)})) & \text{if } \rho_m \neq \infty \\ E(V_Y(y)) & \text{if } \rho_m = \infty \end{cases}$$

Theorem 3.5 *If utilities are monotonically decreasing over the values of the additive value function $V_Y(y)$ with the power-additive utility function given below*

$$U(v) = \begin{cases} K(1 - e^{-((1-V_Y(y))/\rho_m)}) & \text{if } \rho_m \neq \infty \\ 1 - V_Y(y) & \text{if } \rho_m = \infty \end{cases}$$

where $K = 1/(1 - e^{-1/\rho_m})$ and $v = V_Y(y) = \sum_{i=1}^{n} w_i V_{X_i}(x_i)$ then

$$E(U(v)) = \begin{cases} K(1 - E(e^{-((1-V_Y(y))/\rho_m)})) & \text{if } \rho_m \neq \infty \\ 1 - E(V_Y(y)) & \text{if } \rho_m = \infty \end{cases}$$

More on Theorems 3.4 and 3.5

These two theorems provide the way to compute the expected utilities of the power-additive utility function. When computed, these expected utilities provide the measures with which to rank uncertain alternatives, from most-to least-preferred. The following presents a set of formulas needed to compute these expected utilities, when uncertainties are expressed as either discrete or continuous probability distributions.

First, we look at Theorem 3.4. Here, utilities are monotonically increasing over the values of the additive value function. From Theorem 3.4

$$E(U(v)) = \begin{cases} K(1 - E(e^{-(V_Y(y)/\rho_m)})) & \text{if } \rho_m \neq \infty \\ E(V_Y(y)) & \text{if } \rho_m = \infty \end{cases}$$

For the case where $\rho \neq \infty$, the term $E(e^{-(V_Y(y)/\rho_m)})$ can be written as follows:

$$E(e^{-(V_Y(y)/\rho_m)}) = E(e^{-(w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + \cdots + w_n V_{X_n}(x_n))/\rho_m}$$

$$E(e^{-(V_Y(y)/\rho_m)}) = E(e^{-(w_1 V_{X_1}(x_1))/\rho_m}) E(e^{-(w_2 V_{X_2}(x_2))/\rho_m}) \ldots E(e^{-(w_n V_{X_n}(x_n))/\rho_m})$$

where the X_i's are independent random variables and where

$$E(e^{-(w_i V_{X_i}(x_i))/\rho_m}) = \begin{cases} \sum_{x_i} p_{X_i}(x_i) e^{-(w_i V_{X_i}(x_i))/\rho_m} & \text{if } X_i \text{ is discrete} \\ \int_{-\infty}^{\infty} e^{-(w_i V_{X_i}(x_i))/\rho_m} f_{X_i}(x_i)\, dx_i & \text{if } X_i \text{ is continuous} \end{cases} \tag{3.44}$$

Here, $p_{X_i}(x_i)$ is the probability the uncertain outcome X_i takes the score x_i if X_i is a discrete random variable and $f_{X_i}(x_i)$ is the probability density function of X_i if X_i is a continuous random variable. For the case where $\rho = \infty$, in Theorem 3.4, the term $E(V_Y(y))$ can be written as follows:

$$E(V_Y(y)) = w_1 E(V_{X_1}(x_1)) + w_2 E(V_{X_2}(x_2)) + \cdots + w_n E(V_{X_n}(x_n)) \tag{3.45}$$

where

$$E(V_{X_i}(x_i)) = \begin{cases} \sum_{x_i} p_{X_i}(x_i) V_{X_i}(x_i) & \text{if } X_i \text{ is discrete} \\ \int_{-\infty}^{\infty} V_{X_i}(x_i) f_{X_i}(x_i)\, dx_i & \text{if } X_i \text{ is continuous} \end{cases} \tag{3.46}$$

Next, we look at Theorem 3.5. Here, utilities are monotonically decreasing over the values of the additive value function. From Theorem 3.5

$$E(U(v)) = \begin{cases} K(1 - E(e^{-((1 - V_Y(y))/\rho_m)})) & \text{if } \rho_m \neq \infty \\ 1 - E(V_Y(y)) & \text{if } \rho_m = \infty \end{cases}$$

For the case where $\rho \neq \infty$, the term $E(e^{-((1-V_Y(y))/\rho_m)})$ can be written as follows:

$$E(e^{-((1-V_Y(y))/\rho_m)}) = E(e^{-(1-(w_1 V_{X_1}(x_1)+w_2 V_{X_2}(x_2)+\cdots+w_n V_{X_n}(x_n)))/\rho_m})$$
$$E(e^{-((1-V_Y(y))/\rho_m)}) = E(e^{(-1+(w_1 V_{X_1}(x_1)+w_2 V_{X_2}(x_2)+\cdots+w_n V_{X_n}(x_n)))/\rho_m})$$
$$E(e^{-((1-V_Y(y))/\rho_m)}) = e^{-1/\rho_m} E(e^{w_1(V_{X_1}(x_1))/\rho_m}) E(e^{w_1(V_{X_2}(x_2))/\rho_m}) \ldots E(e^{w_n(V_{X_n}(x_n))/\rho_m})$$

where the X_i's are independent random variables and where

$$E(e^{(w_i V_{X_i}(x_i))/\rho_m}) = \begin{cases} \sum_{x_i} p_{X_i}(x_i) e^{(w_i V_{X_i}(x_i))/\rho_m} & \text{if } X_i \text{ is discrete} \\ \int_{-\infty}^{\infty} e^{(w_i V_{X_i}(x_i))/\rho_m} f_{X_i}(x_i)\, dx_i & \text{if } X_i \text{ is continuous} \end{cases} \tag{3.47}$$

In the above, $p_{X_i}(x_i)$ is the probability the uncertain outcome X_i takes the score x_i if X_i is a discrete random variable and $f_{X_i}(x_i)$ is the probability density function of X_i if X_i is a continuous random variable. For the case where $\rho = \infty$, in Theorem 3.5, the term $1 - E(V_Y(y))$ can be written as follows:

$$1 - E(V_Y(y)) = 1 - (w_1 E(V_{X_1}(x_1)) + w_2 E(V_{X_2}(x_2)) + \cdots + w_n E(V_{X_n}(x_n)) \tag{3.48}$$

where

$$E(V_{X_i}(x_i)) = \begin{cases} \sum_{x_i} p_{X_i}(x_i) V_{X_i}(x_i) & \text{if } X_i \text{ is discrete} \\ \int_{-\infty}^{\infty} V_{X_i}(x_i) f_{X_i}(x_i)\, dx_i & \text{if } X_i \text{ is continuous} \end{cases} \tag{3.49}$$

Case Discussion 3.2: Consider the following case. A new and highly sophisticated armored ground transport vehicle is currently being designed. There are three design alternatives undergoing engineering tests and performance trade studies. A set of evaluation criteria to evaluate these designs has been defined by the program's decision-makers. Suppose these criteria are *Operational Days*, *Maintenance/Service Time*, and *Cost* and are denoted by X_1, X_2, and X_3 respectively.

The criterion *Operational Days* refers to the number of days the vehicle can operate without maintenance or servicing. The criterion *Maintenance/Service Time* refers to the number of labor hours needed to service the vehicle to keep it operationally on duty. The criterion *Cost* refers to each vehicle's estimated recurring unit cost in dollars million.

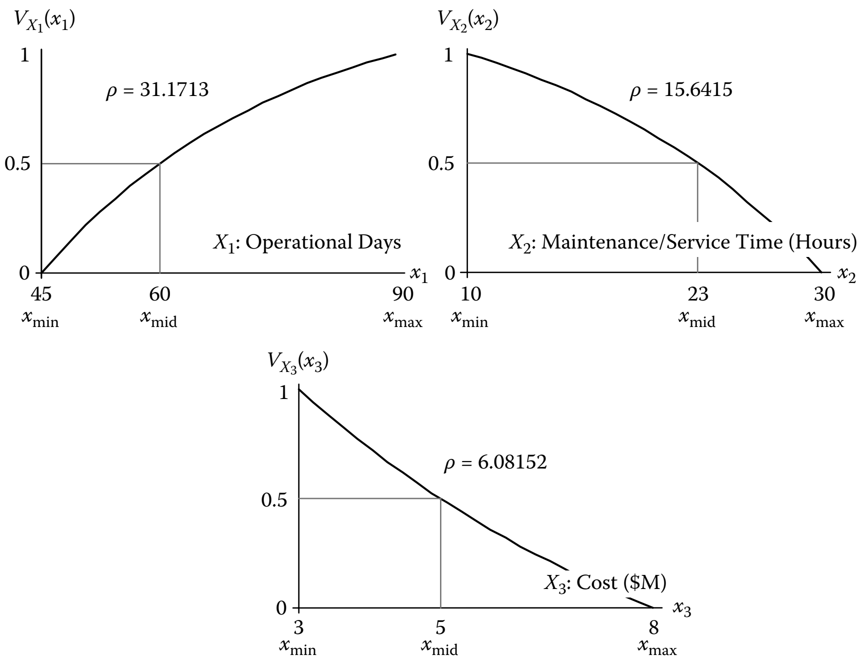


Figure 3.25: Exponential value functions for Case Discussion 3.2.

Suppose these decision-makers assessed the criterion *Operational Days* as twice as important as criterion *Maintenance/Service Time*. Furthermore, suppose they also assessed criterion *Cost* as twice as important as the criterion *Maintenance/Service Time*.

After careful deliberation, suppose the program's decision-makers defined a set of exponential value functions for each of the three criteria. These functions are shown in Figure 3.25.

The equations for these value functions are given below.

$$V_{X_1}(x_1) = 1.30902(1 - e^{0.0320808\,(45-x_1)})$$

$$V_{X_2}(x_2) = 1.38583(1 - e^{0.0639325\,(x_2-30)})$$

$$V_{X_3}(x_3) = -0.784058(1 - e^{-0.164433\,(x_3-8)})$$

Suppose the decision-makers also reviewed the graphs in Figure 3.24 and determined their multiattribute risk tolerance is represented by the curve with

TABLE 3.5: Case Discussion 3.2: Design Alternative Performance Measures

Design Alternative	Criterion X_1 Operational Days	Criterion X_2 Maintenance/ Service Hours	Criterion X_3 Cost ($M)
Alternative A	72–79, $X_1 \sim$ Unif (72, 79)	15–23, $X_2 \sim$ Unif (15, 23)	5.5–7, $X_3 \sim$ Unif (5.5, 7)
Alternative B	85–88, $X_1 \sim$ Unif (85, 88)	23–27, $X_2 \sim$ Unif (23, 27)	5–6.5, $X_3 \sim$ Unif (5, 6.5)
Alternative C	80–85, $X_1 \sim$ Unif (80, 85)	24–28, $X_2 \sim$ Unif (24, 28)	4–5, $X_3 \sim$ Unif (4, 5)

$\rho_m = 0.25$. So, their preference structure reflects a monotonically increasing risk averse attitude over increasing values of the value function.

Suppose each design alternative is undergoing various engineering analyses, cost estimates, and simulations to assess their potential performance on the criteria in Figure 3.25. The results predicted from these analyses are summarized in Table 3.5. Suppose the uncertainties in the outcomes for each criterion are captured by a uniform probability density function — specified for each criterion within a given alternative.

From this information and the data in Table 3.5 determine which design alternative is performing "best," where best is measured as the alternative having the highest expected utility, in terms of the value of each design choice. In this case discussion, assume that conditions for an additive value function hold, as well as utility independence.

Solution to Case Discussion 3.2: To determine which design alternative is performing "best" we will drive toward computing the expected utility of the value of each alternative, as well as computing each alternative's expected value. The alternative with the highest expected utility for value will be considered the "best" among the three design choices.

Since the decision-makers determined their multiattribute risk tolerance is represented by the exponential utility curve with $\rho_m = 0.25$, their preference structure reflects a monotonically increasing risk-averse attitude over increasing values of the value function. Thus, Theorem 3.4 applies. We will use this theorem to determine the expected utility for the value of each design alternative.

Applying Theorem 3.4: Analysis Setup

Since, in this case, $\rho_m = 0.25$ we have from Theorem 3.4

$$E(U(v)) = K(1 - E(e^{-(V_Y(y)/\rho_m)})) \tag{3.50}$$

where $K = 1/(1 - e^{-1/\rho_m})$ and $v = V_Y(y) = \sum_{i=1}^{n} w_i V_{X_i}(x_i)$. Given the parameters in this case, Equation 3.50 becomes

$$E(U(v)) = 1.01865736(1 - E(e^{-4V_Y(y)})) \tag{3.51}$$

where

$$v = V_Y(y) = \frac{2}{5}V_{X_1}(x_1) + \frac{1}{5}V_{X_2}(x_2) + \frac{2}{5}V_{X_3}(x_3) \tag{3.52}$$

and

$$V_{X_1}(x_1) = 1.30902(1 - e^{0.0320808\,(45-x_1)}) \tag{3.53}$$

$$V_{X_2}(x_2) = 1.38583(1 - e^{0.0639325\,(x_2-30)}) \tag{3.54}$$

$$V_{X_3}(x_3) = -0.784058(1 - e^{-0.164433\,(x_3-8)}) \tag{3.55}$$

Next, we will look at the term $E(e^{-4V_Y(y)})$ in Equation 3.51. Here,

$$E(e^{-4V_Y(y)}) = E(e^{-4(\frac{2}{5}V_{X_1}(x_1)+\frac{1}{5}V_{X_2}(x_2)+\frac{2}{5}V_{X_3}(x_3))}) \tag{3.56}$$

If we assume X_1, X_2, and X_3 are independent random variables then

$$E(e^{-4V_Y(y)}) = E(e^{\frac{-8}{5}V_{X_1}(x_1)})E(e^{\frac{-4}{5}V_{X_2}(x_2)})E(e^{\frac{-8}{5}V_{X_3}(x_3)}) \tag{3.57}$$

where

$$E(e^{\frac{-8}{5}V_{X_1}(x_1)}) = \int_{-\infty}^{\infty} e^{\frac{-8}{5}V_{X_1}(x_1)} f_{X_1}(x_1)\, dx_1 \tag{3.58}$$

$$E(e^{\frac{-4}{5}V_{X_2}(x_2)}) = \int_{-\infty}^{\infty} e^{\frac{-4}{5}V_{X_2}(x_2)} f_{X_2}(x_2)\, dx_2 \tag{3.59}$$

$$E(e^{\frac{-8}{5}V_{X_3}(x_3)}) = \int_{-\infty}^{\infty} e^{\frac{-8}{5}V_{X_3}(x_3)} f_{X_3}(x_3)\, dx_3 \tag{3.60}$$

and $f_{X_i}(x_i)$ is the probability density function for X_i which, in this case discussion, is given to be a uniform distribution for each X_i.

Computation Illustration: Computing $E(U(v))$ and $E(v)$ for Design Alternative A

First, compute the value of Equations 3.58 through 3.60 given the parameters in Table 3.5 for Design Alternative A. These computations are given below. The integrals were computed numerically by the application *Mathematica*® [4].

$$E(e^{\frac{-8}{5}V_{X_1}(x_1)}) = \int_{72}^{79} e^{\frac{-8}{5}(1.30902(1-e^{0.0320808\,(45-x_1)}))} \frac{1}{79-72}\, dx_1 = 0.271391$$

$$E(e^{\frac{-4}{5}V_{X_2}(x_2)}) = \int_{15}^{23} e^{\frac{-4}{5}(1.38583(1-e^{0.0639325\,(x_2-30)}))} \frac{1}{23-15}\, dx_2 = 0.57663$$

$$E(e^{\frac{-8}{5}V_{X_3}(x_3)}) = \int_{5.5}^{7} e^{\frac{-8}{5}(-0.784058(1-e^{-0.164433\,(x_3-8)}))} \frac{1}{7-5.5}\, dx_3 = 0.660046$$

Entering these values into Equation 3.57 we have

$$E(e^{-4V_Y(y)}) = E(e^{\frac{-8}{5}V_{X_1}(x_1)})E(e^{\frac{-4}{5}V_{X_2}(x_2)})E(e^{\frac{-8}{5}V_{X_3}(x_3)})$$

$$= (0.2713921)(0.57663)(0.660046) = 0.103292$$

Substituting this value for $E(e^{-4V_Y(y)})$ into Equation 3.51 we have

$$E(U(v)) = 1.01865736(1 - 0.103292) = 0.913438 \sim 0.91$$

Next, we proceed to compute the expected value $E(v)$ for this design alternative. Here, we need to determine $E(v)$ where

$$\begin{aligned} E(v) &= E(V_Y(y)) = E\left(\frac{2}{5}V_{X_1}(x_1) + \frac{1}{5}V_{X_2}(x_2) + \frac{2}{5}V_{X_3}(x_3)\right) \\ &= \frac{2}{5}E(V_{X_1}(x_1)) + \frac{1}{5}E(V_{X_2}(x_2)) + \frac{2}{5}E(V_{X_3}(x_3)) \end{aligned} \tag{3.61}$$

The terms in Equation 3.61 are determined as follows:

$$\begin{aligned} E(V_{X_1}(x_1)) &= \int_{-\infty}^{\infty} V_{X_1}(x_1)\, f_{X_1}(x_1)\, dx_1 \\ &= \int_{72}^{79} 1.30902(1 - e^{0.0320808\,(45-x_1)})\frac{1}{79-72}\, dx_1 = 0.815941 \\ E(V_{X_2}(x_2)) &= \int_{-\infty}^{\infty} V_{X_2}(x_2)\, f_{X_2}(x_2)\, dx_2 \\ &= \int_{15}^{23} 1.38583(1 - e^{0.0639325\,(x_2-30)})\frac{1}{23-15}\, dx_2 = 0.692384 \\ E(V_{X_3}(x_3)) &= \int_{-\infty}^{\infty} V_{X_3}(x_3)\, f_{X_3}(x_3)\, dx_3 \\ &= \int_{5.5}^{7} -0.784058(1 - e^{-0.164433\,(x_3-8)})\frac{1}{7-5.5}\, dx_3 = 0.264084 \end{aligned}$$

Substituting these into Equation 3.61 we have

$$\begin{aligned} E(v) &= E(V_Y(y)) = E\left(\frac{2}{5}V_{X_1}(x_1) + \frac{1}{5}V_{X_2}(x_2) + \frac{2}{5}V_{X_3}(x_3)\right) \\ &= \frac{2}{5}E(V_{X_1}(x_1)) + \frac{1}{5}E(V_{X_2}(x_2)) + \frac{2}{5}E(V_{X_3}(x_3)) \\ &= \frac{2}{5}(0.815941) + \frac{1}{5}(0.692384) + \frac{2}{5}(0.264084) \qquad (3.62) \\ &= 0.5704868 \sim 0.57 \end{aligned}$$

TABLE 3.6: Case Discussion 3.2: Summary Computations

Design Alternative	Criterion X_1 Operational Days	Criterion X_2 Maintenance/ Service Hours	Criterion X_3 Cost ($M)	Expected Value $E(v)$	Expected Utility $E(U(v))$
Alternative A	72–79 $X_1 \sim$ Unif (72, 79)	15–23 $X_2 \sim$ Unif (15, 23)	5.5–7 $X_3 \sim$ Unif (5.5, 7)	0.57	0.91
Alternative B	85–88 $X_1 \sim$ Unif (85, 88)	23–27 $X_2 \sim$ Unif (23, 27)	5–6.5 $X_3 \sim$ Unif (5, 6.5)	0.60	0.93
Alternative C	80–85 $X_1 \sim$ Unif (80, 85)	24–28 $X_2 \sim$ Unif (24, 28)	4–5 $X_3 \sim$ Unif (4, 5)	0.67	0.95

This concludes the computation illustration for Design Alternative A. The same types of computations are performed for the other design alternatives. The results of these computations are summarized in Table 3.6. Design Alternative C is the "best" option in the set. It has the highest expected utility when compared with Alternative A and Alternative B.

3.4 Applications to Engineering Risk Management

A lot of ground has been covered in the preceding sections. An introduction to value and utility functions was presented that included the application of concepts from chapter 2; namely, probability theory, expectation, and random variable distributions. This section offers a further discussion and application of these concepts from an engineering risk management perspective.

In section 3.2, we discussed the concept of value functions and how they are used to assess the "attractiveness" of an alternative among a set of competing alternatives. As we have seen, this assessment is typically made with respect to multiple evaluation criteria. In engineering risk management, risks are analogous to alternatives. Here, management needs to assess the "criticality" of each risk among a set of "competing" risks. This assessment can also be done with respect to multiple evaluation criteria. Figure 3.26 illustrates three common criteria or dimensions of an engineering system commonly impacted by risk.

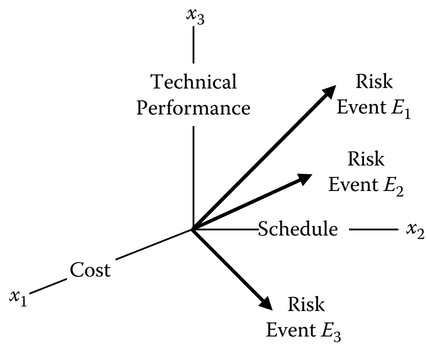


Figure 3.26: Typical risk impact dimensions.

A Value Function Perspective

Let's consider how value functions can be applied in this context. In Figure 3.26, E_1, E_2, and E_3 are risk events that impact an engineering system's *Cost*, *Schedule*, and *Technical Performance*. The risk event with the highest impact is considered the most critical. The risk event with the next highest impact is considered the next most critical and so forth.

Following the approach described in section 3.2, a value function can be specified for each dimension (or criterion) in Figure 3.26. Depending on the nature of the dimension (or criterion), these value functions might be piecewise linear or vary continuously across levels (or scores). Furthermore, some of these dimensions do not have a natural or common unit of measure. In these cases, a constructed scale may be needed.

Consider a risk event's impact on the technical performance of an engineering system. Technical performance is a difficult dimension (or criterion) to express in a common unit. This is because technical performance can be measured in many ways, such as the number of millions of instructions per second or the weight of an end-item. It is difficult, then, to specify for an engineering system a value function for *Technical Performance* along a common measurement scale. A constructed scale is often appropriate in this case.

Figure 3.27 illustrates a piecewise linear value function designed along a constructed scale for the dimension (or criterion) *Technical Performance*. Suppose this function was designed by the engineering system's management team.

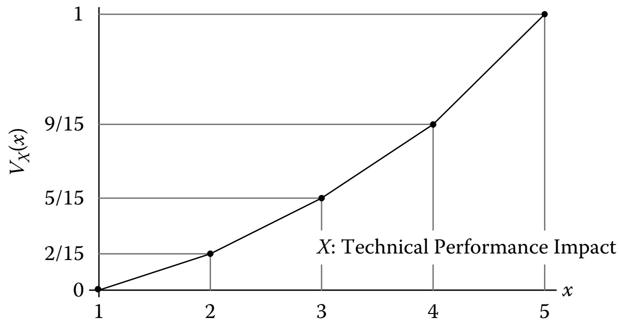


Figure 3.27: A value function for technical performance impact.

There are many ways to define such a constructed scale and its associated value function. Table 3.7 provides one set of linguistic definitions for the levels (scores) corresponding to the constructed scale illustrated in Figure 3.27.

In Figure 3.27, the anchor points 0 and 1 along the vertical axis correspond to level 1 and level 5, respectively, along the horizontal axis. Suppose it was decided the smallest increment Δ in value occurs between a level 1 and level 2 technical performance impact. If we use Δ as the reference standard, it can be seen the team decided the following: (1) the value increment between a level 2 and level 3 technical performance impact is one and a half times the smallest value increment Δ (2) the value increment between a level 3 and level 4 technical performance impact is two times the smallest value increment Δ; and (3) the value increment between a level 4 and level 5 technical performance impact is three times the smallest value increment Δ.

From a risk management perspective, the value function in Figure 3.27 can be interpreted as follows: It reflects a decision-maker's monotonically increasing preferences for risk events that, if they occur, score at increasingly higher levels along the technical performance impact scale. Thus, the higher a risk event scores along the value function, in Figure 3.27, the greater its technical performance impact.

Illustrated in Figure 3.26, a risk event, if it occurs, not only can impact the technical performance of a system but also its cost and schedule. An unmitigated risk may negatively impact the cost of a system, in terms of increased dollars beyond the budget to address problems caused by the risk. In addition, there may also be

TABLE 3.7: A Constructed Scale for Technical Performance Impacts

Ordinal Scale Level (Score)	Definition/Context: Technical Performance Impact
5	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that critical technical performance (or system capability) shortfalls result.
4	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that technical performance (or system capability) is marginally below minimum acceptable levels.
3	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering or these capabilities) to the extent that technical performance (or system capability) falls well-below stated objectives but remains enough above minimum acceptable levels.
2	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that technical performance (or system capability) falls below stated objectives but well-above minimum acceptable levels.
1	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) in a way that results in a negligible effect on overall performance (or achieving capability objectives for a build/block/increment), but regular monitoring for change is strongly recommended.

adverse schedule impacts in terms of missed milestones or schedule slippages beyond what was planned.

To address these concerns, suppose the engineering system's management team designed the two value functions in Figure 3.28. These value functions capture a risk event's impacts on an engineering system's cost and schedule. Here, cost and schedule impacts are shown as single dimensional monotonically increasing exponential value functions.

In designing these value functions, suppose the management team decided a 5% increase in cost and a 3-month increase in schedule to be the midvalues for the cost and schedule value functions, respectively. From Definition 3.1, the general

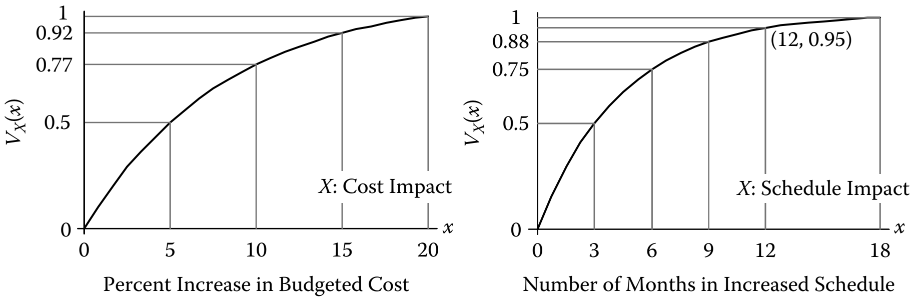


Figure 3.28: Illustrative value functions for cost and schedule impacts.

form of the value functions in Figure 3.28 is given below.

$$V_X(x) = \frac{1 - e^{-(x - x_{\min})/\rho}}{1 - e^{-(x_{\max} - x_{\min})/\rho}}$$

Next, we'll determine the exponential constant ρ for the cost impact value function. Since, in this case $x_{\text{mid}} = 5$ we need to solve the following for ρ.

$$V_X(5) = 0.5 = \frac{1 - e^{-(5-0)/\rho}}{1 - e^{-(20-0)/\rho}} = \frac{1 - e^{-(5)/\rho}}{1 - e^{-(20)/\rho}} \tag{3.63}$$

Solving Equation 3.63 numerically yields $\rho = 8.2$. Thus, the cost impact value function is given by Equation 3.64.

Cost Impact Value Function:

$$V_X(x) = \frac{1 - e^{-x/8.2}}{1 - e^{-20/8.2}} = 1.096(1 - e^{-x/8.2}) \tag{3.64}$$

A similar procedure is used to determine the exponential constant for the schedule impact value function. For this case, it can be computed that $\rho = 4.44$. Thus, the schedule impact value function is given by Equation 3.65.

Schedule Impact Value Function:

$$V_X(x) = \frac{1 - e^{-x/4.44}}{1 - e^{-18/4.44}} = 1.018(1 - e^{-x/4.44}) \tag{3.65}$$

Figure 3.29 presents all three value functions, developed for this discussion.

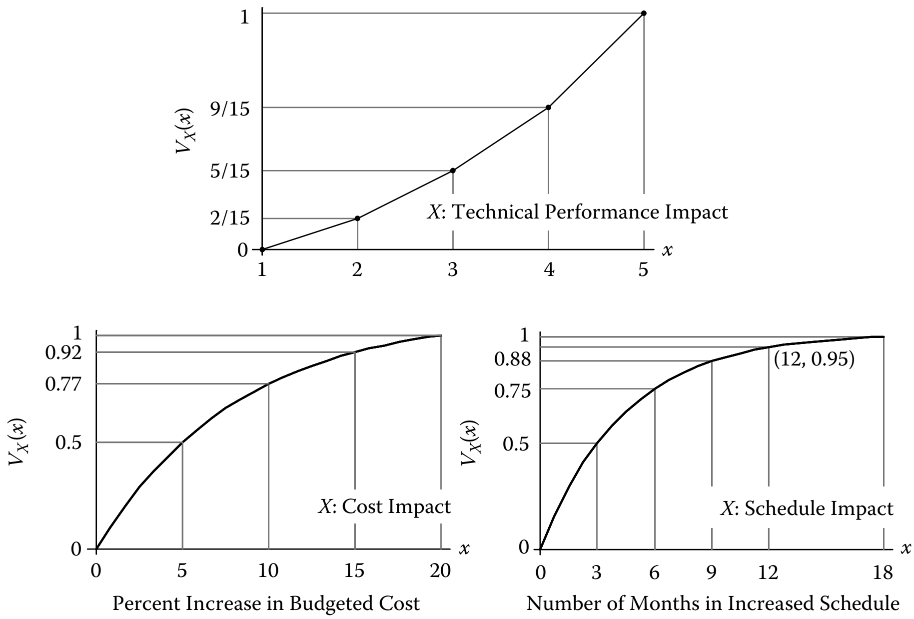


Figure 3.29: Illustrative value functions for a risk event's cost, schedule, and technical performance impacts.

The functions in Figure 3.29 for a risk event's impact on cost and schedule are exponential value functions. These value functions vary continuously across their levels (or scores). It is possible to represent the cost and schedule value functions in an ordinal context. Examples are given in Tables 3.8 and 3.9.

Figure 3.26 illustrated three dimensions of an engineering system commonly impacted by risk. In practice, other dimensions can be impacted by risk. For example, a risk's programmatic impact is often as serious a concern as its impacts on a system's technical performance, its cost, or its schedule.

In this regard, programmatic impacts might refer to specific work products or activities necessary to advance the program along its milestones or its life cycle. Examples of technical work products include system architecture documents, system design documents, the system's engineering management plan, concepts of operation, and the system's logistics plan. Examples of programmatic work products include the system's integrated master schedule, its life cycle cost estimate, its

TABLE 3.8: An Ordinal Scale Representation for Cost Impact

Ordinal Scale Level (Score)	Definition/Context: Cost Impact
5	A risk event that, if it occurs, will cause more than a 15% increase but less than or equal to a 20% increase in the program's budget.
4	A risk event that, if it occurs, will cause more than a 10% increase but less than or equal to a 15% increase in the program's budget.
3	A risk event that, if it occurs, will cause more than a 5% increase but less than or equal to a 10% increase in the program's budget.
2	A risk event that, if it occurs, will cause more than a 2% but less than or equal to a 5% increase in the program's budget.
1	A risk event that, if it occurs, will cause less than a 2% increase in the program's budget.

configuration management plan, its risk management plan, and various acquisition or contracting-related documents and plans.

Figure 3.30 illustrates a value function that could be used to express a risk event's programmatic impacts. Table 3.10 illustrates a constructed scale associated to this value function.

TABLE 3.9: An Ordinal Scale Representation for Schedule Impact

Ordinal Scale Level (Score)	Definition/Context: Schedule Impact
5	A risk event that, if it occurs, will cause more than a 12-month increase in the program's schedule.
4	A risk event that, if it occurs, will cause more than a 9-month but less than or equal to a 12-month increase in the program's schedule.
3	A risk event that, if it occurs, will cause more than a 6-month but less than or equal to a 9-month increase in the program's schedule.
2	A risk event that, if it occurs, will cause more than a 3-month but less than or equal to a 6-month increase in the program's schedule.
1	A risk event that, if it occurs, will cause less than a 3-month increase in the program's schedule.

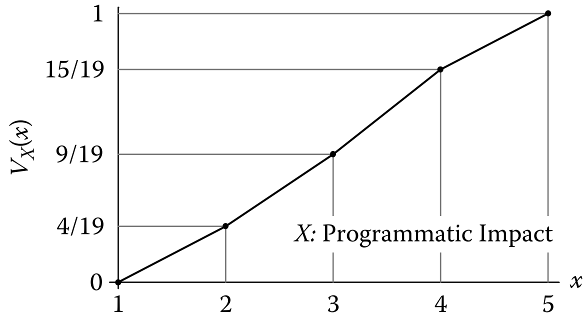


Figure 3.30: Illustrative value function for programmatic impact.

TABLE 3.10: A Constructed Scale for Programmatic Impacts

Ordinal Scale Level (Score)	Definition/Context: Programmatics
5	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more critical objectives for technical or programmatic work products (or activities) will not be completed.
4	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (or activities) is marginally below minimum acceptable levels.
3	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (or activities) falls well-below goals but remains enough above minimum acceptable levels.
2	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (or activities) falls below goals but well-above minimum acceptable levels.
1	A risk event that, if it occurs, has little to no impact on programmatic efforts. Program advancing objectives for technical or programmatic work products (or activities) for a build/block/increment will be met, but regular monitoring for change is strongly recommended.

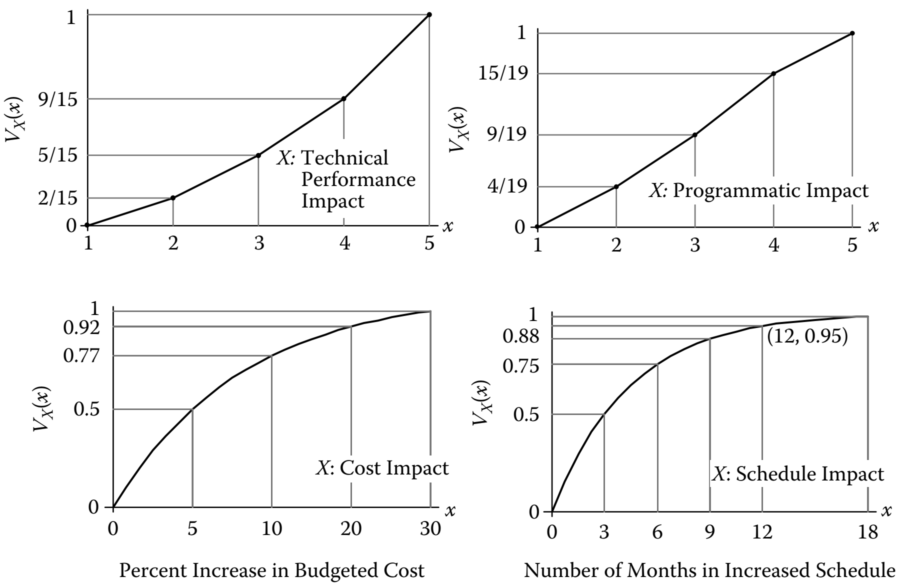


Figure 3.31: A summary set of value functions.

A value scale for programmatic impacts would be developed in a manner similar to the technical performance, cost, and schedule impact scales previously discussed. For example, along the vertical axis, in Figure 3.30, the anchor points 0 and 1 are assigned by the team and correspond to level 1 and level 5, respectively, along the horizontal axis. Suppose it was decided the smallest increment Δ in value occurs between a level 1 and level 2 programmatic impact. If we use Δ as the reference standard, it can be seen the team decided the following: (1) the value increment between a level 2 and level 3 programmatic impact is one and a quarter times the smallest value increment Δ (2) the value increment between a level 3 and level 4 programmatic impact is one and a half times the smallest value increment Δ and (3) the value increment between a level 4 and level 5 programmatic impact is the same as the value increment between a level 1 and level 2 programmatic impact.

Figure 3.31 summarizes the four value functions developed for this section.* We will return to these in Chapter 4.

*The value functions shown in Figure 3.31 are illustrative. In practice a project team must define their own criteria and their specific value functions in a way that truly captures the areas of impact that concern the project team and its management.

A Concluding Thought

A concluding thought on the theory and formalisms presented in this chapter is well stated by R. L. Keeney, research professor of decision sciences, The Fuqua School of Business, Duke University. In his book *Value-Focused Thinking: A Path to Creative Decision Making* [6, page 154], Professor Keeney discusses the question *Are Value Models Scientific or Objective?* He offers the following:

> The final issue concerns the charge that value models are not scientific or objective. With that, I certainly agree in the narrow sense. Indeed values are subjective, but they are undeniably a part of decision situations. Not modeling them does not make them go away. It is simply a question of whether these values get included implicitly and perhaps unknowingly in a decision process or whether there is an attempt to make them explicit and consistent and logical. In a broader sense, the systematic development of a model of values is definitely scientific and objective. It lays out the assumptions on which the model is based, the logic supporting these assumptions, and the basis for data (that is, specific value judgments). This makes it possible to appraise the implications of different value judgments. All of this is very much in the spirit of scientific analysis. It certainly seems more reasonable — even more scientific — to approach important decisions with the relevant values explicit and clarified rather than implicit and vague.

It is in this spirit we extend and apply the formalisms herein, to the very real and complex management problems faced in engineering today's systems.

Questions and Exercises

1. Consider the value function in Figure 3.3. Sketch the value function subject to the following value increments. The smallest value increment occurs between the colors yellow and red; the value increment between red and green is one and a half times the smallest value increment; the value increment between green and blue is two times the smallest value increment; the value increment between blue and black is three times the smallest value increment. Compare and contrast this value function with the value function in Figure 3.3.

2. Consider Figure 3.6. Determine the exponential constant for this value function if the midvalue for the mechanical device's repair time is 15 hours.

3. Review Case Discussion 3.1. Work through the computations.

4. Review and give examples of a nominal scale, an ordinal scale, a cardinal interval scale, a cardinal ratio scale.

5. If a utility function $U(x)$ is monotonically decreasing (i.e., less is better) such that $U(x_{\min}) = 1$ and $U(x_{\max}) = 0$ show that the expected utility is equal to the probability p that $x_{\min}$ occurs.

6. Suppose a lottery X has a range of outcomes bounded by x_1 and x_2. Suppose the probability of any outcome between x_1 and x_2 is uniformly distributed. If $U(x) = a - be^{-x}$, where a and b are constants, show that the certainty equivalent x_{CE} is

$$x_{\text{CE}} = -\ln\left(\frac{e^{-x_1} - e^{-x_2}}{x_2 - x_1}\right)$$

7. Suppose $U(x)$ is a monotonically increasing exponential utility function of the form given in Equation 3.25. Show that the certainty equivalent is given by Equation 3.34.

8. Suppose $U(x) = x^2$ over the interval $0 \leq x \leq 1$ and that $x = 0$ with probability p and $x = 1$ with probability $1 - p$. Show that $E(U(x)) > U(E(X))$. What do you conclude about the risk attitude of this decision-maker?

9. Prove Theorem 3.2. In Theorem 3.2 show that, for $\rho \neq \infty$,

$$E(U(x)) = k(1 - e^{x_{\min}/\rho} E(e^{-x/\rho}) = 1 - p$$

where $k = 1/(1 - e^{-(x_{\max} - x_{\min})/\rho})$

10. Prove Theorem 3.3. In Theorem 3.3 show that, for $\rho \neq \infty$,

$$E(U(x)) = k(1 - e^{-x_{\max}/\rho} E(e^{x/\rho}) = p$$

where $k = 1/(1 - e^{-(x_{\max} - x_{\min})/\rho})$

11. Show that

$$1 - V_{X\text{Inc}}(x, \rho) = V_{X\text{Dec}}(x, -\rho)$$

where $V_{X\mathrm{Inc}}(x, \rho)$ is the increasing exponential value function with parameter ρ given by Equation 3.3 and $V_{X\mathrm{Dec}}(x, -\rho)$ is the decreasing exponential value function with parameter -ρ given by Equation 3.4. Show this general property holds for the power-additive utility functions defined by Equations 3.42 and 3.43.

12. Show that the power-additive utility function is the same for monotonically increasing or decreasing preferences when $v = V_Y(y) = 1/2$.

13. Show that the certainty equivalent for value v_{CE} associated with the power-additive utility function (for monotonically increasing utilities) can be written as

$$v_{\mathrm{CE}} = -\rho_m \ln(1 - E(U(v))/K)$$

where $K = 1/(1 - e^{-1/\rho_m})$.

14. Review Case Discussion 3.2. Work through the computations to the extent practical with available computing software.

15. Compute v_{CE} for each design alternative in Case Discussion 3.2.

References

[1] Keeney, R. L., and H. Raiffa. 1976. *Decisions With Multiple Objectives: Preferences and Value Tradeoffs*, New York: John Wiley & Sons, Inc.

[2] Kirkwood, C. W. 1997. *Strategic Decision Making: Multiobjective Decision Analysis with Spreadsheets*, Pacific Grove, CA: Duxbury Press.

[3] Dyer, J. S., and R. K. Sarin. 1979. "Measurable Multiattribute Value Functions," *Operations Research*, Vol. 27, No. 4, July–August, 1979.

[4] Wolfram, S. 1991. *Mathematica®: A System for Doing Mathematics by Computer*, 2nd ed. Reading, MA: Addison-Wesley Publishing Company, Inc.

[5] Stevens, S. S. 1946. "On the Theory of Scales of Measurement" *Science*, Vol. 103, pp. 677–680.

[6] Keeney, R. L. 1992. *Value-Focused Thinking: A Path to Creative Decision Making*, Cambridge, MA: Harvard University Press.

[7] Clemen, R. T. 1996. *Making Hard Decisions: An Introduction to Decision Analysis*, 2nd ed., Pacific Grove, CA: Brooks/Cole Publishing Company.

[8] von Winterfeldt D., and W. Edwards. 1986. *Decision Analysis and Behavioral Research*, Cambridge UK, Cambridge University Press.

Chapter 4

Analytical Topics in Engineering Risk Management

4.1 Introduction

This chapter presents a series of essays on selected analytical topics that arise in engineering risk management. These topics were chosen due to the frequency with which they occur in practice and because they constitute the basics of any sound risk management process.

Topics in this chapter build upon the analytics presented in the preceding chapters. This includes how to identify, write, and represent risks; methods to rank-order or prioritize risks in terms of their potential impacts to an engineering system project; and, how to monitor progress in managing or mitigating a risk's potential adverse effects. In addition, two current and applied topics in engineering risk management are discussed.

The first topic is technical performance measures and how they can be used to monitor and track an engineering system's overall performance risk. For a system, these measures individually generate useful data; however, little has been developed in the engineering management community on how to integrate them into meaningful measures of performance risk — measures that can be readily tracked over time.

The second topic concludes the chapter with a discussion on risk management in the context of engineering enterprise systems. This is presented from a capability portfolio view. Applying, adapting, or defining risk management principles in an enterprise-wide problem space is at the "cutting edge" of current practice.

4.2 Risk Identification and Approaches

. . . You can manage only the risks identified.

Paul R. Garvey, Book's Author

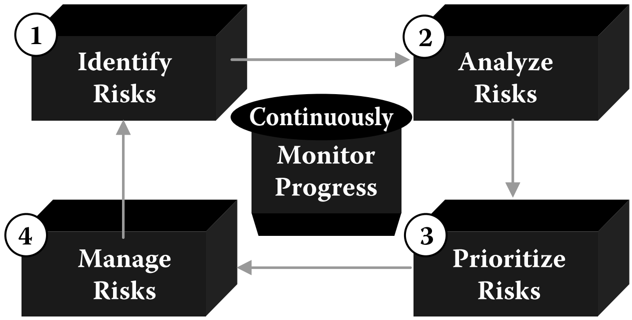


Figure 4.1: A basic view of the risk management process.

This section presents a discussion on ways risks can be identified in an engineering system project. Risk identification is the first and most important step in the risk management process, illustrated in Figure 4.1. Risk identification defines the set of future events that, if any occur, could have unwanted impacts on an engineering system project's cost, schedule, technical performance or any other evaluation criteria defined by the engineering team.

The objective of risk identification is to enumerate known risks and, in so doing, identify risks not immediately evident to the engineering team. As a process, risk identification is a continuous activity that operates regularly throughout the engineering phases of an evolving system.

Risk identification is best performed as a team. Ideally, the senior manager of the project leads this team. Risk identification sessions are often held as a facilitated meeting under the guidance of a professional facilitator. When risks are being identified, it is essential that subject matter experts from all the engineering disciplines participate. This includes staff from the project's cost-schedule team, logistics/supportability team, and the production/manufacturing team.

As a continuous activity, risk identification should be performed by the project team on a regular basis, perhaps biweekly. All stakeholders of the engineering system project have the responsibility to assist in the identification, validation, and eventual resolution of risk.

Inputs to the risk identification process come from many sources. Some sources are particularly relevant to the pre/post-contract award phases of an engineering system project. The content in these sources and materials often provide the basis for a risk and justify why it is a potential concern to an engineering system project. For example, upon review, it may be determined that, pre-contract award, a source document known as a *System Engineering Plan* inadequately addresses information assurance. Identifying this would justify information assurance as a potential technical risk. It might also justify it as a potential schedule risk to the project due to possible delays in system security certification.

Risks can be identified and validated through systematic engineering analyses, such as modeling and simulation, as well as by the application of observation, judgment, and experience. Risk identification efforts include reviews of written materials and interviews with subject experts in specific areas of the project. Working sessions are regularly held with key team members and experienced personnel to review and validate all identified risks.

Throughout the risk identification process, dependencies among risks must also be identified. In this regard, the risk of failing to achieve one objective often impacts the ability to achieve others. Table 4.1 presents a summary of common, but significant, risk areas that can negatively affect an engineering system project.

Table 4.2 presents a set of guidelines for identifying risks associated with an engineering system project. These guidelines are excerpted from the United States Department of Defense *Risk Management Guide*, June 2003 [1].

Writing a Risk: The Condition-If-Then Construct [2, 3]

Here, we revisit this topic from Chapter 2 and further discuss the importance of expressing an identified risk in the *Condition-If-Then* risk statement construct. Mentioned previously, each identified risk should be expressed formally. A "best practice" for writing an identified risk is to follow the *Condition-If-Then* construct. This construct applies in all risk management processes designed for any systems engineering environment. It is based on the recognition that a risk event is, by its nature, a probabilistic event.

What is the *Condition-If-Then* construct? Here, the *Condition* reflects what is known today. It is the *root cause* of the identified risk event. Thus, the *Condition* is an event that has occurred, is presently occurring, or will occur with certainty. Risk events are future events that may occur *because* of the *Condition* present. The following illustrates this construct.

TABLE 4.1: Potential Risk Areas to an Engineering System Project [1]

Area	Significant Risks
Threat	Uncertainty in threat accuracy; sensitivity of design and technology to threat; vulnerability of the system to threat and threat countermeasures; vulnerability to intelligence penetration.
Requirements	Performance requirements not properly established; requirements not stable; required operating environment not described; requirements do not address logistics and sustainability; lack of user or stakeholder participation in requirements definition.
Design	Design implications not sufficiently considered in concept exploration; system will not satisfy user requirements; mismatch of system design solutions to user needs; human-machine interface problems; increased skills or training requirements identified late in the acquisition process; design not cost effective; design relies on immature technologies or "exotic" materials to achieve performance objectives; software design, coding, and testing not adequately planned or resourced.
Test and Evaluation	Test planning not initiated early in the project; testing does not address the ultimate operating environment; test procedures do not address all major performance and suitability specifications; test facilities not available to accomplish specific tests, especially system-level tests; insufficient time to test thoroughly.
Modeling and Simulation (M&S)	M&S tools or technologies are not verified, validated, or accredited for the intended purpose; project lacks proper analysis tools and modeling and simulation capability or technologies to assess the current design or identified alternatives.

TABLE 4.1: Potential Risk Areas to an Engineering System Project [1] (*Continued*)

Area	Significant Risks
Technology	Project depends on unproven technology for success — there are no defined technology alternatives; project success depends on achieving advances in state-of-the-art technology; potential advances in technology will result in less than optimal costs or make system components obsolete; technology has not been demonstrated in required operating environment; technology relies on complex hardware, software, or integration design.
Logistics	Inadequate supportability late in development or after fielding resulting in need for engineering changes, increased costs, and/or schedule delays; life cycle costs not accurate because of poor logistics supportability analyses; logistics analyses results not included in cost-performance tradeoffs; design trade studies do not include supportability considerations.
Production/ Facilities	Production implications not considered during concept exploration; production not sufficiently considered during design; inadequate planning for long lead items and vendor support; production processes not proven; prime contractors do not have adequate plans for managing subcontractors; facilities not readily available for cost-effective production; contract offers no incentive to modernize facilities or reduce cost.
Concurrency	Immature or unproven technologies will not be adequately developed before production; production funding will be available too early — before development effort has sufficiently matured; concurrency established without clear understanding of risks.
Technical Capability of Developer	Developer has limited experience in specific type of development; contractor has poor track record relative to costs and schedule; contractor experiences loss of key personnel; prime contractor relies excessively on subcontractors for major development efforts.

(*Continued*)

TABLE 4.1: Potential Risk Areas to an Engineering System Project [1] (*Continued*)

Area	Significant Risks
Cost, Funding, Schedule	Cost-schedule objectives not realistic; cost-schedule estimates do not reflect true program uncertainties; cost-schedule-performance tradeoffs not done; unstable requirements prevent establishing a cost-schedule baseline; funding profiles do not match acquisition strategy (or plan) across annual budget cycles.
Acquisition and Program Management	Acquisition strategy understates true program challenges (e.g., performance, technology maturity, cost-schedule uncertainties, viability of industrial base, economic stability); alternative acquisition strategies or program management options not considered or planned; inability to staff program management team with essential skill sets; risk management not performed or not effective or results ignored; none or inadequate socialization with (or engagement by) users/stakeholders in key technical or program milestones (e.g., requirements definition, design reviews, operational testing, etc).

Suppose we have the following two events. Define the *Condition* as event B and the *If* as event A (the risk event)

$$B = \{\text{Current test plans are focused on the components of the subsystem and not on the subsystem as a whole}\}$$

$$A = \{\text{Subsystem will not be fully tested when integrated into the system for full-up system-level testing}\}$$

The risk event is the *Condition-If* part of the construct; specifically,

Risk Event: {The subsystem will not be fully tested when integrated into the system for full-up system-level testing, because current test plans are focused on the components of the subsystem and not on the subsystem as a whole}

TABLE 4.2: Some Guidelines for Identifying Risks [1]

Step	Guidelines
1	Understand the requirements and the project's performance goals, which are typically defined as thresholds and objectives. Understand the operational (functional and environmental) conditions under which these values must be achieved.
2	Determine technical and performance risks related to engineering and manufacturing processes. Identify those processes that are planned or needed to design, develop, produce, support, and retire the system. Compare these processes with industry best practices and identify variances or new, untried, processes. These variances or untried processes are sources of risk. The contractor should review the processes to be used by its subcontractors to ensure they are consistent with best industry practices.
3	Determine technical and performance risks associated with the engineering system project and all its subsystems (e.g., a communications subsystem) to include the following critical risk areas: design and engineering, technology, logistics, supportability, concurrency, and manufacturing.
4	Ensure cost-schedule objectives are realistic and cost-schedule estimates reflect true program uncertainties; identify whether cost-schedule-performance options exist that offer less risk but still meet user needs; work to baseline requirements and that users/stakeholders have been engaged; ensure funding profiles match acquisition strategy (or planning) across annual budget cycles.
5	All identified risks are documented in a risk management database, with a statement of the risk and a description of the conditions or root cause(s) generating the concern and the context of the risk.

Mathematically, a risk event is equivalent to a probability event. Formally,

$$0 < P(A \mid B) = \alpha < 1$$

where α is the probability risk event A occurs given the conditioning event B (the root cause event) has occurred.

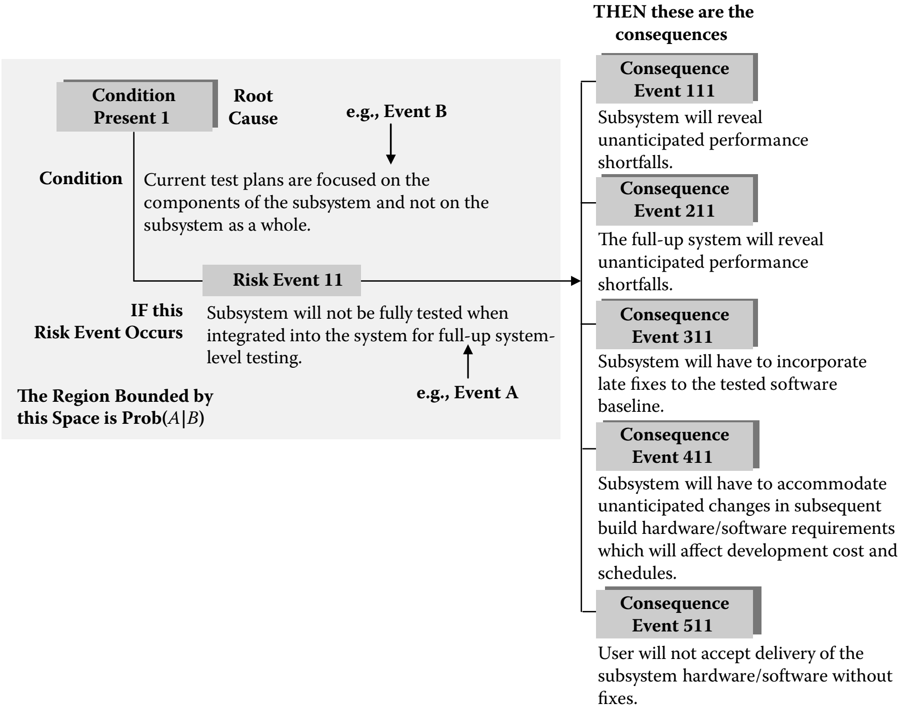


Figure 4.2: The Risk Statement: An illustration of the *Condition-If-Then* construct [3].

In summary, a "best practice" protocol for writing a risk is to follow the *Condition-If-Then* risk statement construct. Here, the *Condition* is as described above (i.e., it is the root cause). The *If* is the associated risk event. The *Then* is the consequence, or set of consequences, that will impact the engineering system project if the risk event occurs. An example of a risk statement written in the *Condition-If-Then* construct is illustrated in Figure 4.2.

4.3 Risk Analysis and Risk Prioritization

. . . Take calculated risks; that is quite different from being rash.

U.S. Army Gen. George S. Patton, (1885–1945)

This section presents ways risk events can be analyzed and prioritized. Risk events are analyzed in terms of their occurrence probabilities and potential impacts to an engineering system project. Risks are prioritized in terms of establishing a most- to least-critical importance ranking. Ranking risks in terms of their criticality or importance provides insights to the project's management on where resources may be needed to manage, or mitigate, potentially high impact/high consequence risk events. The following presents methods for analyzing risks in terms of their occurrence probabilities and potential impacts or consequences to an engineering system project. Algorithms for prioritizing risks in terms of establishing a most-to least-critical importance ranking are provided.

4.3.1 Ordinal Approaches and the Borda Algorithm

Mentioned previously, risk events are analyzed in terms of their *potential* impacts or consequences to an engineering system project. Risks, then, are a function F of their occurrence probabilities and impacts (or consequences) as represented by Equation 4.1.

$$Risk = F(Probability,\ Impact) \tag{4.1}$$

Recognizing this, analyzing and prioritizing risks must take probability and impact (consequence) into account, regardless of whether ordinal or value function formalisms are used.

Ordinal approaches to risk analysis and prioritization are based on procedures that "bin" risks into consequence and probability priority categories. Ordinal procedures are a valid but high-level way to analyze and rank-order risk events.

Ordinal approaches are based on the development of ordinal scales for a risk event's impact (consequence) and occurrence probability. Recall from Chapter 3, an ordinal scale is a measurement scale in which attributes are assigned a number that represents order. Also, recall this is order or rank *only*. That is, the distance between values in an ordinal scale is indeterminate — from this, it follows that arithmetic operations on ordinal numbers *are not permissible*.

The Ordinal Risk Matrix

The ordinal risk matrix is a widely used high-level approach for "binning" risk events into priority (or criticality) categories. Figure 4.3 presents a classical 5 × 5 ordinal risk matrix. Here, two ordinal scales define the matrix. These are the

Probability Level					
5	1,5	2,5	3,5	4,5	5,5
4	1,4	2,4	3,4	4,4	5,4
3	1,3	2,3	3,3	4,3	5,3
2	1,2	2,2	3,2	4,2	5,2
1	1,1	2,1	3,1	4,1	5,1
	1	2	3	4	5

Impact (Consequence) Level

Figure 4.3: A traditional 5×5 ordinal risk matrix.

scales for *Probability Level* along the vertical side of the matrix and *Impact* (*Consequence*) *Level* along the horizontal side of the matrix.

The higher the probability level the more likely the occurrence of the risk event. The lower the probability level the less likely the occurrence of the risk event. A similar relationship holds for consequence. The higher the impact level the greater the risk event's consequence to the project. The lower the impact level the lesser the risk event's consequence to the project.

A risk event is "binned" into one of the squares of the matrix as a function of the level of its impact (consequence) and the level of its occurrence probability. Denote an (i, j) risk event as one that has a level i impact (consequence) and a level j occurrence probability, where $i, j = 1, 2, 3, 4,$ or 5. How then might risks be ranked or prioritized across the squares of the risk matrix?

A common approach to prioritizing risk events across the squares of the risk matrix is to multiply the impact and probability levels that define each square and use the resultant product to define the square's score. The right-most matrix in Figure 4.4 shows this result. Risk events binned into a specific square receive that square's score and are ranked accordingly. Here, risk events with higher scores have higher priority than risk events with lower scores.

The first problem with this approach is the multiplication of ordinal numbers, which is *not a permissible arithmetic operation*. Because of the first problem, a second problem is the resultant values (or scores) for the squares shown in the right-most matrix of Figure 4.4. Seen above, a risk event with a level 5

Probability Level					
5	1,5	2,5	3,5	4,5	5,5
4	1,4	2,4	3,4	4,4	5,4
3	1,3	2,3	3,3	4,3	5,3
2	1,2	2,2	3,2	4,2	5,2
1	1,1	2,1	3,1	4,1	5,1
Impact (Consequence) Level	1	2	3	4	5

Probability Level					
5	5	10	15	20	25
4	4	8	12	16	20
3	3	6	9	12	15
2	2	4	6	8	10
1	1	2	3	4	5
Impact (Consequence) Level	1	2	3	4	5

Figure 4.4: A common approach to ranking risks in a risk matrix.

impact and a level 1 probability (a (5, 1) risk event) receives the same score as a risk event with a level 1 impact and a level 5 probability (a (1, 5) risk event). These are two very different risk events yet, under this ranking approach, they are equally valued and tie in their scores. In practice, decision-makers should not loose visibility into the *presence* of higher consequence risk events regardless of their occurrence probabilities, especially since these probabilities have a history underestimation.

Observe there are numerous ties in the scores in the right-most risk matrix in Figure 4.4. Again, this leaves the same problem just discussed for the decision-maker. Should risk events that tie in their scores be equally valued, especially when these ties occur in very different consequence-probability regions of the risk matrix? How should ties be broken for purposes of rank-ordering risks? Should impact (consequence), probability, or some combination of the two be the basis for breaking ties? What is the right tradeoff? There are no easy answers to these questions, except, in short, that an *impact times probability* approach for ranking risks *within an ordinal risk matrix* should not be used. What, then, is a way to proceed in an ordinal context?

One way is to first examine the risk attitude of the project team. Is the team impact averse or probability averse? A strictly impact averse team is one not willing to tradeoff consequence for probability. For this team, low probability high consequence risk events should be ranked high enough so they remain visible to management (i.e., they don't fall off the "radar screen"). This may indeed be appropriate for engineering system projects where loss of life consequences are possible.

Probability Level					
5	1,5	2,5	3,5	4,5	5,5
4	1,4	2,4	3,4	4,4	5,4
3	1,3	2,3	3,3	4,3	5,3
2	1,2	2,2	3,2	4,2	5,2
1	1,1	2,1	3,1	4,1	5,1
	1	2	3	4	5
	Impact (Consequence) Level				

Probability Level					
5	21	16	11	6	1
4	22	17	12	7	2
3	23	18	13	8	3
2	24	19	14	9	4
1	25	20	15	10	5
	1	2	3	4	5
	Impact (Consequence) Level				

Figure 4.5: A strictly impact (consequence) averse risk matrix.

In the engineering of systems, most project teams are impact (consequence) averse to varying degrees. What varies from project to project are (1) the strength of the team's consequence averseness and (2) the regions of the risk matrix where impact and probability tradeoffs are *acceptable* to the team. How can risk rankings or risk prioritizations be assigned in these contexts? The following presents one way to address these considerations.

First, redefine the right-most risk matrix in Figure 4.4 in a way that rank-orders (or prioritizes) risk events along a strictly impact (consequence) averse track. This is shown by the right-most risk matrix in Figure 4.5.

Let's look more closely at this matrix. Each square is scored (or marked) not on the basis of multiplying impact by probability but are scored (or marked) directly in order of consequence criticality by the engineering team. Here, the lower the score the higher the priority. In the right-most matrix in Figure 4.5, a (5, 5) risk event falls in the square marked one. Risk events that fall in this square have been assigned the first (or highest) priority. In this matrix, risk events with a level 5 impact (consequence) will always have higher priority than those with a level 4 impact (consequence). Similarly, risk events with a level 4 impact (consequence) will always have higher priority than those with a level 3 impact (consequence) and so forth.

The columns in the right-most risk matrix in Figure 4.5 are first directly ranked by impact (consequence). Then, the squares within each column are directly ranked by probability. Thus, this matrix is one where risk events with the highest impact (consequence) to an engineering system project will always fall into one of the first five squares — and into a specific square as a function of their judged occurrence

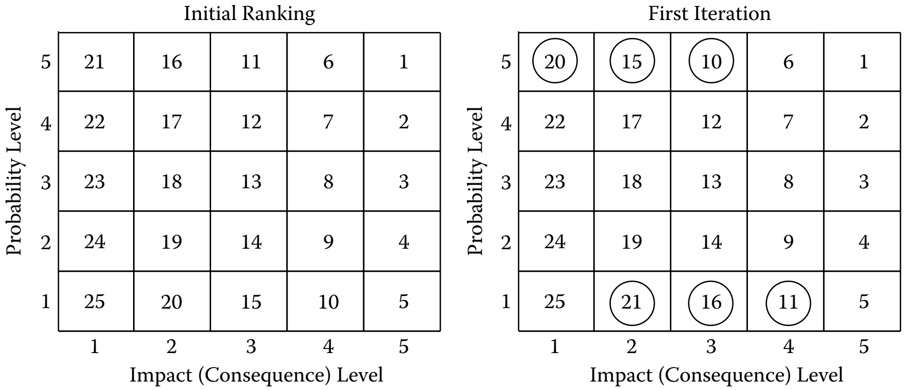


Figure 4.6: First iteration risk ranking.

probability. Next, we look for tradeoffs within this matrix that might be made by a team, in terms of consequence versus probability.

An engineering team might want to relax portions of the strictly consequence averse risk matrix in Figure 4.5. To illustrate this, suppose the team decided any risk event with a level 5 impact (consequence) should remain ranked in one of the first five squares of the matrix. This is the right-most column of the risk matrix. For all other columns, tradeoffs could be made between the bottom square of a right-hand column and the top square of its adjacent left column. This is shown by the circled squares in the matrix in Figure 4.6. Here, let's suppose the engineering team decided a (3, 5) risk event has higher priority than a (4, 1) risk event; a (2, 5) risk event has higher priority than a (3, 1) risk event; a (1, 5) risk event has higher priority than a (2, 1) risk event.

From this first iteration, suppose the engineering team then decided to further refine the rank-ordering of the squares in that matrix. Let's call this the team's second and final iteration. This iteration is shown by the right-most matrix in Figure 4.7. Here, the engineering team decided a (1, 5) risk event or a (1, 4) risk event should have higher priorities than a (2, 2) risk event or a (2, 1) risk event.

This discussion illustrates one approach for directly ranking (or prioritizing) risk events on the basis of their impacts and occurrence probabilities, within the structure of an *ordinal risk matrix*. Other patterns are possible; however, rankings and tradeoff opportunities within an ordinal risk matrix should always first reflect the risk attitude of the engineering system's project team.

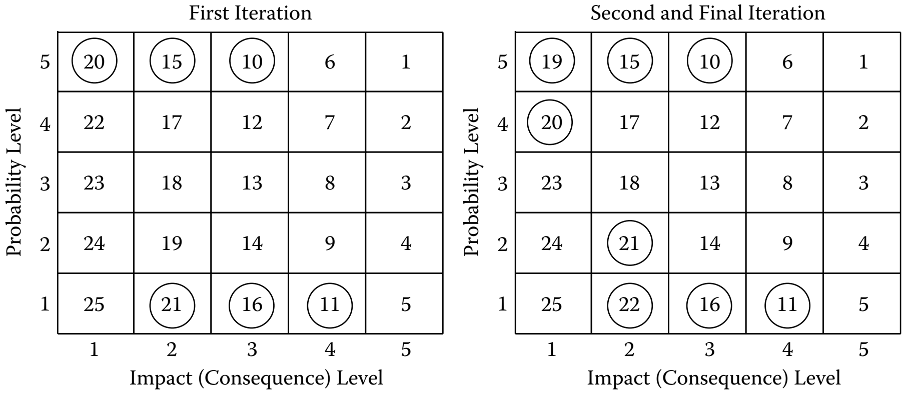


Figure 4.7: Second and final risk ranking iteration.

Finally, it is common practice to assign color bands within a risk matrix. These bands are intended to reflect priority (or criticality) groups within the matrix. Figure 4.8 illustrates how the right-most matrix in Figure 4.7 might be colored with respect to priority (or criticality) groups.

In Figure 4.8 we have the following assigned priority groups. Risk events that fall in the black colored squares are in the first (highest) priority group. Risk events that fall in the dark-red colored squares are in the second priority group. Risk events that fall in the red colored squares are in the third priority group. Risk events that fall in the orange colored squares are in the fourth priority group. Risk events that fall in the yellow colored squares are in the fifth priority group. Risk events that fall in the green colored square are in the last (or sixth) priority group.

Probability Level					
5	Orange 19	Red 15	Dark Red 10	Dark Red 6	Black 1
4	Orange 20	Orange 17	Red 12	Dark Red 7	Black 2
3	Yellow 23	Orange 18	Red 13	Dark Red 8	Black 3
2	Yellow 24	Yellow 21	Red 14	Dark Red 9	Black 4
1	Green 25	Yellow 22	Orange 16	Red 11	Black 5
	1	2	3	4	5
	Impact (Consequence) Level				

Figure 4.8: A priority (or criticality) colored risk matrix.

Priority Group

- ❶ Black: Squares 1–5
- ❷ Dark Red: Squares 6–10
- ❸ Red: Squares 11–15
- ❹ Orange: Squares 16–20
- ❺ Yellow: Squares 21–24
- ❻ Green: Square 25

❻ ≺ ❺ ≺ ❹ ≺ ❸ ≺ ❷ ≺ ❶

Priority Group Ordering

Figure 4.9: Priority group ordering in Figure 4.8.

Figure 4.9 shows symbolically this ordering relationship. Note the symbol "≺" is used to identify the importance or priority order of a color group.

Working with an Ordinal Risk Matrix: Defining Probability and Impact Levels

The ordinal risk matrix discussed above is defined along ordinal scales for a risk event's occurrence probability and impact (consequence). When ordinal scales are used in this context, probability and impact (consequence) need only be linguistically defined by increasing levels of severity.

In Chapter 3, a set of constructed ordinal scales was presented for a risk event's impact on cost, schedule, technical performance, and programmatics. What about probability? What might a constructed ordinal scale look like for a risk event's occurrence probability? An example of such a scale is presented in Table 4.3 [2].

Table 4.3 derives from an actual engineering system project. Five levels are shown. The higher the level the greater the chance a risk event will occur. The number of levels and their linguistic interpretations must be tailored to reflect the risk attitude of the engineering team, conditions within which the system is being engineered, and the environment within which the system will operate.

The next step in working with an ordinal risk matrix is defining the impact (consequence) scale. In Chapter 3, four constructed scales were presented to assess a risk event's impact on an engineering system project. These were scales for cost,

TABLE 4.3: An Ordinal Scale for Occurrence Probability [2]

Ordinal Scale Level (Score)	Definition/Context: Occurrence Probability
Very High (VH) 5	e.g., a risk event that has an occurrence probability greater than 0.85 but less than one (a certainty). e.g., from "*Almost Sure to Occur*" to "*Extremely Sure to Occur*"
High (H) 4	e.g., a risk event that has an occurrence probability greater than 0.65 but less than or equal to 0.85. e.g., from "*Likely to Occur*" to "*Very Likely to Occur*"
Moderate (M) 3	e.g., a risk event that has an occurrence probability greater than 0.35 but less than or equal to 0.65. e.g., from "*Somewhat Less Than an Even Chance to Occur*" to "*Somewhat Greater Than an Even Chance to Occur*"
Low (L) 2	e.g., a risk event that has an occurrence probability greater than 0.15 but less than or equal to 0.35. e.g., from "*Not Very Likely to Occur*" to "*Not Likely to Occur*"
Very Low (VL) 1	e.g., a risk event that has an occurrence probability greater than zero but less than or equal to 0.15. e.g., from "*Extremely Sure Not to Occur*" to "*Almost Sure Not to Occur*"

schedule, technical performance, and programmatic impacts. We'll refer to these same scales to illustrate their use in an ordinal risk matrix. For convenience they are shown again on the following pages.

Once probability and impact (consequence) scales have been defined by the engineering team, each identified risk event is evaluated and rated against the levels in these scales. Here, there is only one table for probability; so, for this scale, the level assessed for a risk event's occurrence probability is directly mapped to the vertical axis of the risk matrix. What about impact? There are four of these tables. How do you determine an overall impact (consequence) level?

One approach is to take the maximum impact (consequence) level from across the four evaluation tables; that is, define a risk event's overall impact level as follows:

$$\text{Overall Impact Level} = \text{Max}\,\{\text{Cost Level, Schedule Level, Technical Performance Level, Programmatic Level}\}$$

TABLE 4.4: An Ordinal Scale for Technical Performance Impact

Ordinal Scale Level (Score)	Definition/Context: Technical Performance Impact
5	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that critical technical performance (or system capability) shortfalls result.
4	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that technical performance (or system capability) is marginally below minimum acceptable levels.
3	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering or these capabilities) to the extent that technical performance (or system capability) falls well-below stated objectives but remains enough above minimum acceptable levels.
2	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) to the extent that technical performance (or system capability) falls below stated objectives but well-above minimum acceptable levels.
1	A risk event that, if it occurs, impacts the system's operational capabilities (or the engineering of these capabilities) in a way that results in a negligible effect on overall performance (or achieving capability objectives for a build/block/increment), but regular monitoring for change is strongly recommended.

Table 4.8 illustrates this for four risk events. Each event is also mapped into a square and a color band defined by the risk matrix in Figure 4.8. Figure 4.10 (shown farther ahead) presents a combined risk matrix view of these four events.

From Table 4.8 we can infer the following:

$$\textit{Risk Event } \#3 \prec \textit{Risk Event } \#4 \prec \textit{Risk Event } \#1 \prec \textit{Risk Event } \#2$$

TABLE 4.5: An Ordinal Scale Representation for Cost Impact

Ordinal Scale Level (Score)	Definition/Context: Cost Impact
5	A risk event that, if it occurs, will cause more than a 15% increase but less than or equal to a 20% increase in the program's budget.
4	A risk event that, if it occurs, will cause more than a 10% increase but less than or equal to a 15% increase in the program's budget.
3	A risk event that, if it occurs, will cause more than a 5% increase but less than or equal to a 10% increase in the program's budget.
2	A risk event that, if it occurs, will cause more than a 2% but less than or equal to a 5% increase in the program's budget.
1	A risk event that, if it occurs, will cause less than a 2% increase in the program's budget.

TABLE 4.6: An Ordinal Scale Representation for Schedule Impact

Ordinal Scale Level (Score)	Definition/Context: Schedule Impact
5	A risk event that, if it occurs, will cause more than a 12-month increase in the program's schedule.
4	A risk event that, if it occurs, will cause more than a 9-month but less than or equal to a 12-month increase in the program's schedule.
3	A risk event that, if it occurs, will cause more than a 6-month but less than or equal to a 9-month increase in the program's schedule.
2	A risk event that, if it occurs, will cause more than a 3-month but less than or equal to a 6-month increase in the program's schedule.
1	A risk event that, if it occurs, will cause less than a 3-month increase in the program's schedule.

TABLE 4.7: An Ordinal Scale for Programmatic Impact

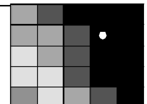
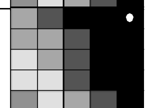
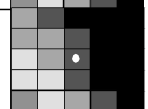
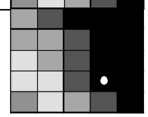
Ordinal Scale Level (Score)	Definition/Context: Programmatics
5	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more critical objectives for technical or programmatic work products (or activities) will not be completed.
4	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (or activities) is marginally below minimum acceptable levels.
3	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (or activities) falls well below goals but remains enough above minimum acceptable levels.
2	A risk event that, if it occurs, impacts programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (or activities) falls below goals but well above minimum acceptable levels.
1	A risk event that, if it occurs, has little to no impact on programmatic efforts. Program advancing objectives for technical or programmatic work products (or activities) for a build/block/increment will be met, but regular monitoring for change is strongly recommended.

In Table 4.8, risk event #2 falls into the highest priority square. Risk event #1 has second priority. Risk event #4 has third priority. Risk event #3 has fourth priority.

The above illustrated an ordinal risk matrix that involved only four risk events. Observe these events all fell into separate squares of the risk matrix. Thus, a clear ordinal-based priority ranking is present, in this case.

In practice, dozens of risk events are typically identified on a project. In these cases, it is common for risk events to "collect" or "bunch-up" into certain squares within the risk matrix. This signals the presence of ties. That is, risk events that "collect" or "bunch-up" into the same square tie in their priority ordering.

TABLE 4.8: An Ordinal Risk Matrix: Hypothetical Set of Risk Events

Risk Event #	Probability Level	Impact (or Consequence) Level: Cost	Schedule	Technical Performance	Programmatics	Risk Matrix
1	4	2	3	4	3	
		Maximum Impact (or Consequence) Level:			**4**	
2	5	2	2	1	5	
		Maximum Impact (or Consequence) Level:			**5**	
3	3	2	2	3	1	
		Maximum Impact (or Consequence) Level:			**3**	
4	2	3	3	1	4	
		Maximum Impact (or Consequence) Level:			**4**	

The presence of ties in a square is a common characteristic of the ordinal risk matrix. They are usually dealt with outside the risk matrix by the engineering team.

Finally, statistics can be defined and collected on the data that enters a risk matrix. For example, the number of risk events that fall into a specific square of the risk matrix can be tracked. The impact category (e.g., cost, schedule, technical performance, and programmatics) that most frequently drives maximum impact (consequence) levels can be identified and monitored. The distribution and density of risk events across the squares and color bands of the risk matrix can be derived and tracked. These and other statistics can be defined, measured, and monitored as aids in prioritizing risks and deciding where risk mitigating resources are most indicated on the project.

A Frequency Count Approach

Here, we introduce an improvement to an ordinal risk matrix. We refer to this as the *Frequency Count Approach*.* This approach involves counting the number of times specific ordinal levels characterize a risk event across its set of evaluation criteria. For example, a risk event might be evaluated across four consequence criteria — with each criterion defined along an ordinal scale. Suppose this scale

*The *Frequency Count Approach* was created by Dr. Richard A. Moynihan, The MITRE Corporation, 2006.

ranges from level 1–Negligible to level 5–Severe. In the frequency count approach, the number of times a risk event is characterized by a specific ordinal level (e.g., 5–Severe) is counted across the set of evaluation criteria. These counts are then composed into a real number. This number can then be used to ordinal rank each risk in the set of evaluated risk events.

The frequency count approach applies in cases where ordinal level assessments are made across multiple evaluation criteria. Here, one can count the number of times a level 5 impact, for example, has been assessed across a risk event's consequence criteria. When frequency counts across consequence criteria are composed into a real number, we call this number the risk event's *Consequence Code*. Tables 4.8A and 4.8B illustrate this idea. Both tables are equivalent.

Table 4.8A presents 15 risk events along with their occurrence probabilities and consequence ratings. Suppose Table 4.3 was used to assess each event's occurrence probability. Suppose Table 4.4 through Table 4.7 were used to assess each risk event's consequence level on cost, schedule, technical performance, and programmatics.

TABLE 4.8A: Probability and Consequence: Ordinal Level Assessments

Risk Event ID #	Probability Assessment		Consequence Areas & Assessments			
	Cardinal	Ordinal	Cost	Schedule	Technical Performance	Programmatics
1	0.95	5	5	5	5	4
2	0.50	3	2	2	1	5
3	0.25	2	5	5	5	5
4	0.65	3	3	3	1	4
5	0.35	2	3	4	4	5
6	0.95	5	4	4	1	4
7	0.75	4	4	3	4	2
8	0.90	5	5	4	1	4
9	0.45	3	4	5	5	4
10	0.75	4	5	3	5	3
11	0.85	4	1	3	3	4
12	0.25	2	3	5	5	2
13	0.35	2	4	3	4	4
14	0.50	3	5	5	1	3
15	0.75	4	1	3	5	4

TABLE 4.8B: An Equivalent Linguistic Representation of Table 4.8A

Risk Event ID #	Probability Rating Assessment	Consequence Areas & Rating Assessments			
		Cost	Schedule	Technical Performance	Programmatics
1	VH	Severe	Severe	Severe	Significant
2	M	Minor	Minor	Negligible	Severe
3	L	Severe	Severe	Severe	Severe
4	M	Moderate	Moderate	Negligible	Significant
5	L	Moderate	Significant	Significant	Severe
6	VH	Significant	Significant	Negligible	Significant
7	H	Significant	Moderate	Significant	Minor
8	VH	Severe	Significant	Negligible	Significant
9	M	Significant	Severe	Severe	Significant
10	H	Severe	Moderate	Severe	Moderate
11	H	Negligible	Moderate	Moderate	Significant
12	L	Moderate	Severe	Severe	Minor
13	L	Significant	Moderate	Significant	Significant
14	M	Severe	Severe	Negligible	Moderate
15	H	Negligible	Moderate	Severe	Significant

Mentioned previously, Table 4.8A and Table 4.8B are equivalent. The frequency count approach has the flexibility to work with ordinal values (Table 4.8A) or with their equivalent "linguistic" representations (Table 4.8B). This is a choice available to the risk management or engineering team.

From the assessments in Table 4.8A (or Table 4.8B) we compute each risk event's *Consequence Code*. Computing the consequence code for Risk Event #1 will be shown. The consequence codes for the remaining risk events in Table 4.8A (or Table 4.8B) are computed in a similar manner.

From Table 4.8A (or Table 4.8B) observe that Risk Event #1 has a Level 5 impact on three consequence criteria and a Level 4 impact on one consequence criterion. From these, we form the following frequency count.

Risk Event ID #	Consequence Rating Count				
	Level 5	**Level 4**	**Level 3**	**Level 2**	**Level 1**
1	3	1	0	0	0

TABLE 4.8C: Consequence Codes: All 15 Risk Events

	Consequence Rating Count					
Risk Event ID #	**Level 5**	**Level 4**	**Level 3**	**Level 2**	**Level 1**	**Consequence Code**
1	3	1	0	0	0	31,000
2	1	0	0	2	1	10,021
3	4	0	0	0	0	40,000
4	0	1	2	0	1	1,201
5	1	2	1	0	0	12,100
6	0	3	0	0	1	3,001
7	0	2	1	1	0	2,110
8	1	2	0	0	1	12,001
9	2	2	0	0	0	22,000
10	2	0	2	0	0	20,200
11	0	1	2	0	1	1,201
12	2	0	1	1	0	20,110
13	0	3	1	0	0	3,100
14	2	0	1	0	1	20,101
15	1	1	1	0	1	11,101

These counts are then composed into a real number called the *Consequence Code*.*
Specifically, the consequence code for Risk Event #1 is

$$\begin{aligned} \textit{Consequence Code} &= 3\,(10{,}000) + 1\,(1{,}000) + 0\,(100) + 0\,(10) + 0\,(1) \\ &= 31000 \end{aligned}$$

The consequence codes for the entire set of risk events in Table 4.8A (or Table 4.B) are shown in Table 4.8C.

From these data, various displays can be produced to view probability versus consequence code. Figure 4.10A illustrates one view. Figure 4.10B illustrates another. Here, only the ordinal value for probability level is shown along the vertical axis. The value next to each point on these figures is the risk event identification number.

*In this case, we cannot exceed nine consequence criteria. Why? If more than nine are needed, then *Consequence Code* is formed by multiplying the rating count frequencies for levels 5, 4, 3, 2, and 1 by 10^8, 10^6, 10^4, 10^2, and 10^0, respectively. However, in most situations, nine or fewer consequence criteria are sufficient.

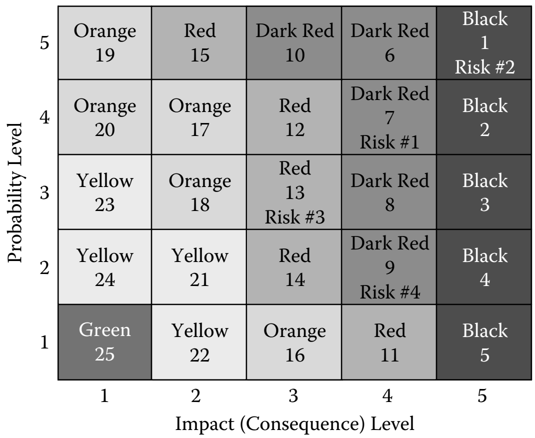


Figure 4.10: Risk events #1 through #4: combined view.

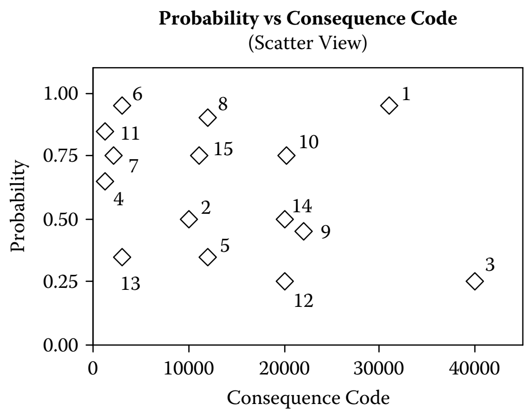


Figure 4.10A: Probability vs. consequence code.

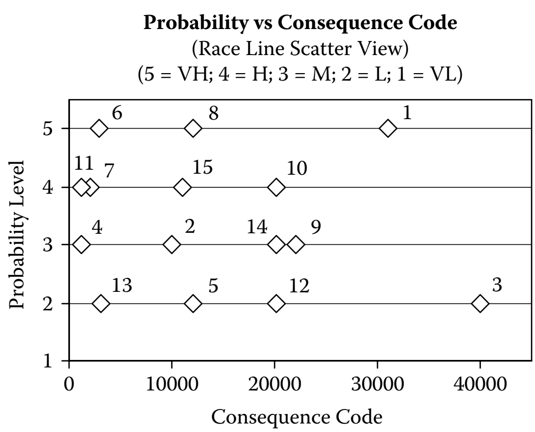


Figure 4.10B: Probability level vs. consequence code.

Probability Level	1	2	3	4	5
5	Orange 19	Red 15	Dark Red 10	Dark Red 6 Risk #6	Black 1 Risk #1, #8
4	Orange 20	Orange 17	Red 12	Dark Red 7 Risk#7, #11	Black 2 Risk#10, #15
3	Yellow 23	Orange 18	Red 13 Risk #3	Dark Red 8 Risk #4	Risk #2 Black 3 Risk #9, #14
2	Yellow 24	Yellow 21	Red 14	Dark Red 9 Risk #13	Risk #3 Black 4 Risk #5, #12
1	Green 25	Yellow 22	Orange 16	Red 11	Black 5

Impact (Consequence) Level
Risk Events "Binned" by Their Maximum Consequence Levels

Figure 4.10C: An ordinal risk matrix mapping of Table 4.8A risks.

The frequency count approach can be used in combination with the ordinal risk matrix. Consider Figure 4.10C. Here, all 15 risk events (in Table 4.8A) are mapped into one of 25 bins, according to their occurrence probabilities and consequence assessments. A risk event's consequence bin is determined by its maximum consequence level across the four criteria shown in Table 4.8A.

In Figure 4.10C, two risk events fall into bin 1. From an ordinal risk matrix perspective, risk event 1 and risk event 8 have the same priority. They share the same probability and consequence levels. However, since a consequence code was computed for each event, it can be used to discriminate between them. This is an improvement to the traditional ordinal risk matrix. It offers a way to break ties between risk events that fall into the same bin of the risk matrix. In this case, the consequence code for risk event 1 was 31000. The consequence code for risk event 8 was 12001. So, within bin 1, risk event 1 is more critical than risk event 8. It has the higher consequence code (in this bin) and should be given the higher priority.

Recall that the ordinal risk matrix above came from an earlier discussion, where a team defined bin priorities by weighting the importance of probability versus consequence. Although different teams may develop different bin priorities, once established the frequency count approach can operate within them.

Finally, it is important to remember that the frequency count approach is also an ordinal scheme. As such, the difference in the consequence code values between risk events does not have meaning. Only whether such values are greater than, less than, or equal to each other have meaning. Risk events with higher consequence codes have greater consequence criticality than those with lower consequence codes — but that is all you can say. The magnitudes of their differences are indeterminate.

The Borda Algorithm

This section introduces a well-known algorithm for developing an ordinal ranking of preferences and illustrates its application in a risk management context. The algorithm is known as the Borda algorithm [4]. It was developed in the late 18th century by the French mathematician Jean-Charles Chevalier de Borda (1733 to 1799), and used to elect members of the French Academy of Sciences.

The Borda algorithm can be found in the literature on voting theory. It has been studied extensively by mathematicians and those who work in social choice theory. The algorithm is classified as a positional voting system because the rank position of each candidate on a ballot is worth a fixed number of points. The algorithm works as follows:

All candidates for an election are ranked by each voter on his or her ballot. If there are n candidates in the election, then the first-place candidate on the ballot receives $(n - 1)$ points, the second-place candidate receives $(n - 2)$ points, the third-place candidate receives $(n - 3)$ points and so forth. The candidate ranked last receives 0 points. In general, the candidate in the ith-place on the ballot receives $(n - i)$ points. The points are summed across all voter ballots and the candidate with the most points wins the election.

For example, suppose we have 10 voters and each ranked his or her preferences for five candidates A, B, C, D, and E. Suppose their rank-orders are as follows:

Voter 1 : A > B > D > C > E

Voter 2 : D > A > B > E > C

Voter 3 : B > A > E > C > D

Voter 4 : A > B > D > E > C

Voter 5 : D > A > C > B > E

Voter 6 : B > A > E > C > D

Voter 7 : A > B > D > E > C

Voter 8 : A > B > D > C > E

Voter 9 : D > E > A > B > C

Voter 10 : A > C > B > D > E

From these 10 ballots, which of the $n = 5$ candidates wins the election? Which candidate is in second place, third place, and so forth? In the above, the symbol > means "preferred to;" that is, A *preferred to* B is written A > B. Lets take a look at the first voter's ballot. Here, A is in first place so A receives $(n - 1) = 4$ points; B is in second place so B receives $(n - 2) = 3$ points; D is in third place so D receives $(n - 3) = 2$ points; C is in fourth place so C receives $(n - 4) = 1$ point; E is in fifth place (last) so E receives $(n - 5) = 0$ points. The same procedure for allocating points is applied to the other nine ballots. The result is shown in Table 4.9.

From Table 4.9, we see that candidate A has the most points and wins the election. The overall preference order across all 10 voters is as follows:

$$A > B > D > E > C$$

The points shown at the bottom of Table 4.9 are referred to as the *Borda Counts*. Notice the total Borda Count for this election is 100 points. The total Borda count for this election can be computed as follows:

$$(\textit{Number of Voters})\,[(n-1)+(n-2)+(n-3)+(n-4)+(n-5)]$$
$$= (10)(5n-15) = 50(n-3) = 50(2) = 100 \text{ since } n = 5 \text{ candidates}$$

TABLE 4.9: Borda Algorithm: Borda Count Tally

	Candidate				
Voter	**A**	**B**	**C**	**D**	**E**
1	4	3	1	2	0
2	3	2	0	4	1
3	3	4	1	0	2
4	4	3	0	2	1
5	3	1	2	4	0
6	3	4	1	0	2
7	4	3	0	2	1
8	4	3	1	2	0
9	2	1	0	4	3
10	4	2	3	1	0
Total	34	26	9	21	10

Next, we will apply this concept to risk management problems involving the ranking of risks, as a function of each risk's occurrence probability and impact criteria. The analogy works this way. Instead of voters we have criteria. Instead of candidates we have risk events. How do we apply the Borda algorithm in this context?

Consider the four risk events in Table 4.8. Each risk was evaluated in terms of its occurrence probability and impact (consequence) according to the ordinal levels defined in Tables 4.3 through 4.7. From these assessments, which risk event is ranked first? How should the remaining risk events be ranked relative to the first?

Here, we have five criteria and $n = 4$ risk events. From the criteria assessments in Table 4.8, the rank-order positions for risk event 1 (R1), risk event 2 (R2), risk event 3 (R3), and risk event 4 (R4) are as follows:

Criterion Probability: R2 > R1 > R3 > R4
Criterion Cost: R4 > R1 = R2 = R3
Criterion Schedule: R1 = R4 > R2 = R3
Criterion Technical Perf: R1 > R3 > R2 = R4
Criterion Programmatics: R2 > R4 > R1 > R3

For the criterion Probability we have R2 in first place so it receives $(n-1)$ points, or 3 points in this case. R1 is in second place so it receives $(n-2)$ points, or 2 points in this case. R3 is in third place so it receives $(n-3)$ points, or 1 point in this case. R4 is in last place so it receives $(n-4)$ points, or 0 points in this case.

For the criterion Cost we have R4 in first place so it receives $(n-1)$ points, or 3 points in this case. R1, R2, and R3 are tied. When ties occur, points allocated to these positions are derived from the average; that is, R1, R2, and R3 each will receive $((n-2)+(n-3)+(n-4))/3$ points in this case. Since $n=4$, R1, R2, and R3 will each receive $((2+1+0))/3 = 1$ point.

For the criterion Schedule, R1 and R4 are tied; hence, they each receive $((n-1)$ $+(n-2))/2 = 2.5$ points in this case. Here, we also have R2 and R3 tied; they each receive $((n-3)+(n-4))/2 = 0.5$ points. This same type of process is applied to the remaining criteria Technical Performance and Programmatics. The resulting point distribution is summarized in Table 4.10.

From Table 4.10, we see that risk event 1 (R1) has the highest Borda count and, therefore, ranks first. The rank-order of all four risk events across the five criteria

TABLE 4.10: Borda Algorithm: Borda Count Tally

	Risk Events			
Criteria	**R1**	**R2**	**R3**	**R4**
Probability	2	3	1	0
Cost	1	1	1	3
Schedule	2.5	0.5	0.5	2.5
Technical Perf.	3	0.5	2	0.5
Programmatics	1	3	0	2
Total	9.5	8	4.5	8

is as follows:

$$R1 > R2 = R4 > R3$$

Left as a discussion topic, how is this risk ranking different from that determined by the risk matrix approach? Why should it be different? The Borda algorithm is an excellent ordinal ranking scheme and offers a number of advantages over other schemes, such as the risk matrix. Discuss some of these advantages.

4.3.2 A Value Function Approach

This section presents a value function approach for analyzing and prioritizing an engineering system project's risks. This is in contrast to the ordinal approaches just discussed. Chapter 3 introduced the general theory of value functions as background for demonstrating their application in a risk management context.

Recall that a *value function* is a real-valued mathematical function defined over an evaluation criterion that represents an option's measure of "goodness" over the levels of the criterion. A measure of "goodness" reflects a decision-maker's judged value in the performance of an option across the levels of a criterion. In this context, an option (or alternative) is treated as a risk event. Value functions are designed to enable expressing the severity of each risk event against a set of evaluation criteria, such as a project's cost, schedule, or technical performance.

Discussed in Chapter 3, a value function is often designed to vary from zero to one over the range of levels (or scores) of a criterion. The value functions in Section 3.4 were designed this way, where risk events falling in the upper-end of a function's range are of greater concern to a project than those falling in the lower-end of its range.

Value functions can also be used to define a risk matrix. Here, the probability and impact (consequence) axes are cardinal scales instead of ordinal scales. This is illustrated in Figure 4.11. Figure 4.11 also shows a scatter plot of four risk events. Since the axes of this risk matrix are cardinal scales, inferences can be made on the magnitude of the differences between the risks shown in this figure.

Mentioned above, section 3.4 illustrated the design of value functions as a way to quantitatively express the severity of a risk event's impact (consequence) on an engineering system project. In particular, section 3.4 developed constructed scales and their associated value functions for capturing the impacts of risk across multiple evaluation criteria. A project's cost, schedule, technical performance, and programmatic impacts were the criteria specified. Although these are common criteria, a project needs to specify criteria and their associated evaluation scales specific to the project's unique issues and constraints.

For this section, we will revisit the value functions in section 3.4 and illustrate their use in the context of analyzing risk events and visualizing them in a cardinal

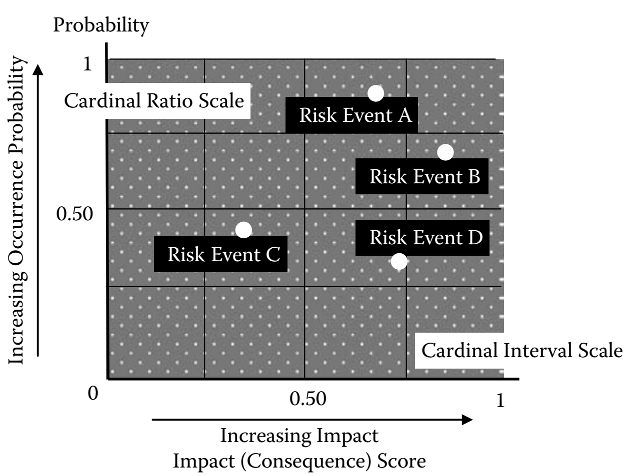


Figure 4.11: A risk matrix defined by cardinal scales.

risk matrix. In addition, we will apply value functions as a way to generate a most-to least-critical cardinal ordering of identified risk events.

Value Functions for Risk Event Impact Areas

Suppose an engineering system's project team identified criteria against which the impacts (or consequences) of risk events would be evaluated. Suppose the criteria are a project's budgeted cost, its development schedule, the technical performance of the system, and the extent risk events impact programmatic considerations. Furthermore, suppose the scales and their associated value functions in section 3.4 are used to represent these criteria. In addition, suppose the project team decided to use the constructed scale in Table 4.3 to guide their direct (subjective) assessment of a risk event's occurrence probability. With this, we will illustrate the use of these scales and value functions to "score" risk events and generate a visualization of them in a cardinal risk matrix.

Figure 4.12 presents value functions designed in section 3.4 for expressing the severity of a risk event's impact (or consequences) on an engineering system's technical performance and programmatic dimensions. Here, two piecewise linear value functions were designed along the constructed scales given by Table 3.7 (or Table 4.4) and Table 3.10 (or Table 4.7), respectively.

Figure 4.13 presents the value functions designed in section 3.4 for expressing the severity of a risk event's impact (or consequences) on an engineering system's budgeted cost and schedule dimensions. Here, cost and schedule impacts are single dimensional monotonically increasing exponential value functions.

In designing these value functions, suppose the management team decided a 5% increase in cost and a 3-month increase in schedule to be the midvalues for the cost

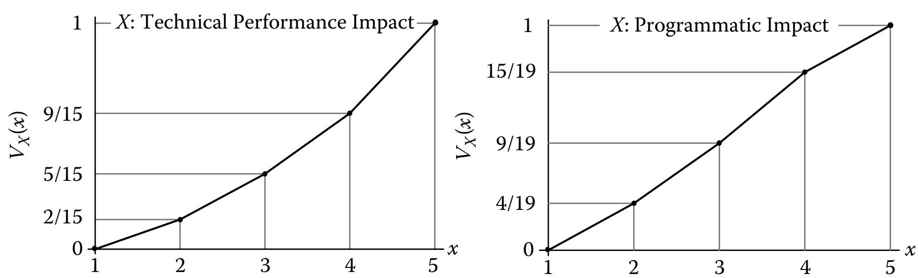


Figure 4.12: Value functions for technical and programmatic impacts.

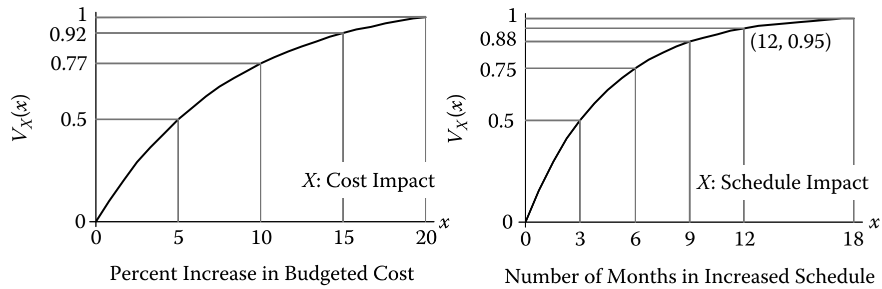


Figure 4.13: Value functions for cost and schedule impacts.

and schedule value functions, respectively. From this, it follows that the formulas for these value functions are given by Equations 4.2 and 4.3, respectively.

$$\text{Cost Impact Value Function: } V_X(x) = 1.096(1 - e^{-x/8.2}) \tag{4.2}$$

$$\text{Schedule Impact Value Function: } V_X(x) = 1.018(1 - e^{-x/4.44}) \tag{4.3}$$

There are many ways to combine the above single dimensional value functions into an overall measure of impact. The following discusses the first of these ways; specifically, the additive value function (Definition 3.6) will be used as one way to derive this measure, which will be called the overall impact score.

Combining Value Functions

The additive value function will be illustrated as one way to compute a risk event's overall impact (consequence) score. For this, assume Definition 3.6 applies.

A value function $V_Y(y)$ is an additive value function if there exists n single dimensional value functions $V_{X_1}(x_1)$, $V_{X_2}(x_2)$, $V_{X_3}(x_3)$, . . . , $V_{X_n}(x_n)$ satisfying

$$V_Y(y) = w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + w_3 V_{X_3}(x_3) + \cdots + w_n V_{X_n}(x_n) \tag{4.4}$$

where w_i for $i = 1, \ldots, n$ are non-negative weights (importance weights) whose values range between zero and one and where $w_1 + w_2 + w_3 + \cdots + w_n = 1$.

From a risk analysis perspective, define $V_{Impact}(E)$ to be an additive value function that measures the overall impact score of risk event E, where

$$V_{Impact}(E) = w_1 V_{Cost}(x_1) + w_2 V_{Sched}(x_2) + w_3 V_{TPerf}(x_3) + w_4 V_{Prgm}(x_4) \tag{4.5}$$

and w_i for $i = 1, \ldots, 4$ are non-negative weights (importance weights) whose values range between zero and one and where $w_1 + w_2 + w_3 + w_4 = 1$.

In Equation 4.5, let $V_{Cost}(x_1)$, $V_{Sched}(x_2)$, $V_{TPerf}(x_3)$, and $V_{Prgm}(x_4)$ denote the single dimensional value functions for a risk event's impact (consequence) on a project's cost, schedule, technical performance, and programmatic areas, given in Figure 4.12 and Figure 4.13.*

Case Discussion 4.1 Consider a satellite communication system that interfaces to a number of networked subsystems. Suppose a data management architecture is being newly designed for the communication system as whole, one where the interfacing subsystem databases must support information exchanges. However, due to schedule pressures suppose the new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released. Suppose the project team identified the following risk event *"Inadequate synchronization of the communication system's new database with the existing subsystem databases."*

Here, two events are described. Let the *Condition* be event B and the *If* be event A; that is,

B = {The new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released.}

A = {Inadequate synchronization of the communication system's new database with the existing subsystem databases.}

From this we can form the risk event, as given below and shown in Figure 4.14.

Risk Event: {Inadequate synchronization of the communication system's new database with the existing subsystem databases, because the new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released.}

*Equation 4.5 is not limited to the evaluation criteria shown. Though they are typical, in practice a project team must define their own criteria and their specific value functions in a way that truly captures the areas of impact that concern the project and its management.

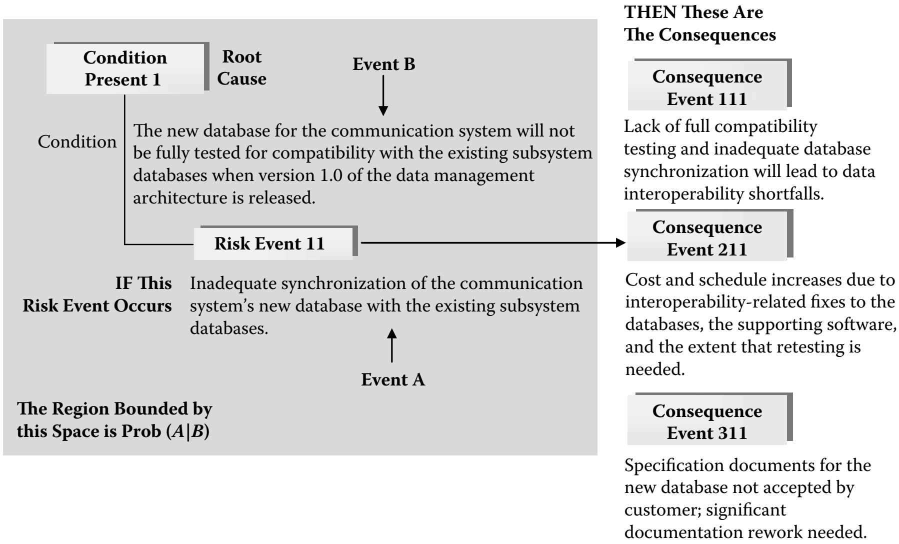


Figure 4.14: Case discussion 4.1 risk statement.

Next, recall that a risk event is equivalent to a probability event; formally,

$$0 < P(A|B) = \alpha < 1$$

where α is the probability risk event A occurs given the conditioning event B (the root cause event) has occurred.

Using Table 4.3 as a guide, suppose the project's engineering team determined inadequate synchronization, at some levels, between the exchange of data across the system's subsystems is almost sure to occur, if the present condition concerning the lack of full compatibility testing persists. To reflect this, suppose they assign the value 0.95 as a judgmental assessment of the probability this risk event will occur; that is,

$$0 < P(A|B) = \alpha = 0.95 < 1$$

Impact (Consequence) Assessment

Next, suppose the engineering team identified three consequence events to the project (shown in Figure 4.14) that will occur, *if risk event A occurs*. Suppose these impacts were assessed, by the team, against the project's cost, schedule, technical performance, and programmatics. Suppose the engineering team documented their *basis of assessment* (BOA) in Table 4.11. In Table 4.11, the ordinal scale level values shown derive from Tables 4.4 through 4.7.

Assessing Importance Weights

The above discussion provides inputs to apply Equation 4.5 as one way to compute an overall impact score of risk event A. Before this can be done, importance weights for the evaluation criteria cost, schedule, technical performance, and programmatics must be determined.

From Equation 4.5, the overall impact score of risk event A is

$$V_{Impact}(A) = w_1 V_{Cost}(x_1) + w_2 V_{Sched}(x_2) + w_3 V_{TPerf}(x_3) + w_4 V_{Prgm}(x_4)$$

where w_i for $i = 1, \ldots, 4$ are non-negative weights (importance weights) for the cost, schedule, technical performance, and programmatics criteria. Recall these weights sum to one; that is,

$$w_1 + w_2 + w_3 + w_4 = 1$$

For this case, suppose the engineering team made the following importance weight assessments. Technical performance w_3 is twice as important as cost w_1; cost w_1 is twice as important as schedule w_2; cost w_1 is twice as important as programmatics w_4. From this, we have the following:

$$w_3 = 2w_1; \quad w_1 = 2w_2; \quad w_1 = 2w_4$$

Since $w_1 + w_2 + w_3 + w_4 = 1$ it follows, from the above relationships, that

$$w_1 + \frac{1}{2}w_1 + 2w_1 + \frac{1}{2}w_1 = 1$$

thus, $w_1 = \frac{1}{4}$. From this, $w_2 = \frac{1}{8}$, $w_3 = \frac{1}{2}$, and $w_4 = \frac{1}{8}$. Substituting these values into Equation 4.5 we have, for this case,

$$V_{Impact}(A) = \frac{1}{4}V_{Cost}(x_1) + \frac{1}{8}V_{Sched}(x_2) + \frac{1}{2}V_{TPerf}(x_3) + \frac{1}{8}V_{Prgm}(x_4) \quad (4.6)$$

In Table 4.11, we have the values for the other terms in Equation 4.6; specifically,

$$V_{Impact}(A) = \frac{1}{4}(0.842) + \frac{1}{8}(0.604) + \frac{1}{2}(0.60) + \frac{1}{8}(0.79) = 0.685 \quad (4.7)$$

TABLE 4.11: Illustrative Impact (Consequence) Assessments and Scores

Impact Assessments	**Basis of Assessment (BOA)**
Ordinal Scale Level (Score)	Risk Event *Inadequate synchronization of the communication system's new database with the existing subsystem databases, because the new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released.* The consequent event descriptions in Figure 4.14 provide a starting point for the basis of assessments below. They support the team's judgments and supporting arguments for articulating the risk event's consequences, if it occurs, on the project's cost, schedule, programmatics, and the system's technical performance.
Cost Impact Level 4	This risk event, if it occurs, is estimated by the engineering team to cause a 12 percent increase in the project's current budget. The estimate is based on a careful assessment of the ripple effects across the project's cost categories for interoperability-related fixes to the databases, the supporting software, and the extent that re-testing is needed. Value Function Value: From Equation 4.2, $V_X(12) = 1.096(1 - e^{-12/8.2}) = 0.842$
Schedule Impact Level 3	This risk event, if it occurs, is estimated by the engineering team to cause a 4 month increase in the project's current schedule. The estimate is based on a careful assessment of the ripple effects across the project's integrated master schedule for interoperability-related fixes to the databases, the supporting software, and the extent that re-testing is needed.

TABLE 4.11: Illustrative Impact (Consequence) Assessments and Scores (*Continued*)

Impact Assessments	Basis of Assessment (BOA)
	Value Function Value: From Equation 4.3, $V_X(4) = 1.018(1 - e^{-4/4.44}) = 0.604$
Technical Performance Impact Level 4	This risk event, if it occurs, is assessed by the engineering team as one that will impact the system's operational capabilities to the extent that technical performance is marginally below minimum acceptable levels, depending on the location and extent of interoperability shortfalls.
	Value Function Value: From Figure 4.12, $V_X(4) = 9/15 = 0.60$
Programmatic Impact Level 4	This risk event, if it occurs, is assessed by the engineering team as one that will impact programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (e.g., various specifications or activities) is marginally below minimum acceptable levels.
	Value Function Value: From Figure 4.12, $V_X(4) = 15/19 = 0.79$

Figure 4.15 shows a plot of this risk event — one with an assessed occurrence probability of 0.95 and an overall impact (consequence) score of 0.685 (from Equation 4.7). Overall, this might be considered a risk of a moderately high concern. This concludes Case Discussion 4.1.

The analysis approach presented in Case Discussion 4.1 can be extended to multiple risk events. This is illustrated in Figure 4.16. Shown is a scatter plot of 25 risk events. Here, each event is analyzed by the same process just discussed. Each risk event is then plotted by its occurrence probability and its overall impact (consequence) to the project. Figures 4.15 and 4.16 illustrate what is meant by a cardinal form of a risk matrix.

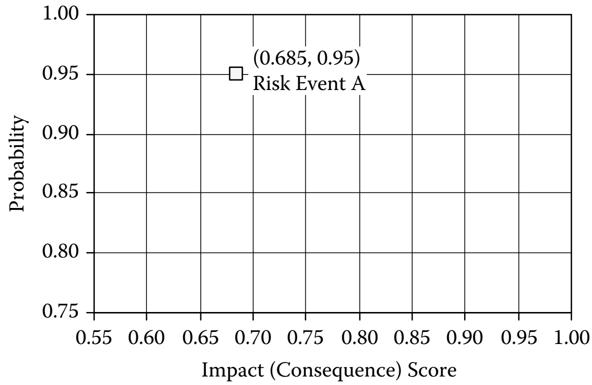


Figure 4.15: Case Discussion 4.1: a plot of risk event A.

In Figure 4.16, risk events 3, 5, and 8 appear to be ahead of the others in terms of occurrence probability and impact (consequence) to the project. What about risk events 24, 4, and 1? How important are they relative to risk events 3, 5, and 8? The following discusses an approach based on the preceding discussion for assessing the *relative rank-order* of risk events, when these events are presented in terms of a cardinal risk matrix or scatter plot.

An Algorithm for Ranking Risk Events

One way to develop a relative rank-order of risk events from information in a cardinal scatter plot is to apply the formulation shown in Equation 4.8. Equation 4.8

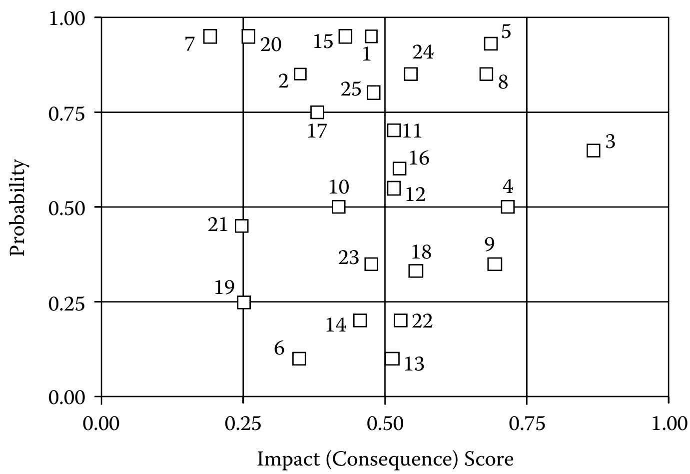


Figure 4.16: A scatter plot of 25 risk events.

is an additive value function. Here, we define the risk score of risk event E by

$$Risk\ Score\,(E) = u_1 Prob\,(E) + u_2 V_{Impact}\,(E) \tag{4.8}$$

where coefficients u_1 and u_2 are non-negative weights that sum to one, the first term is a value function for the event's occurrence probability, and the second term is a value function for the event's overall impact on the project; that is,

$$V_{Impact}\,(E) = w_1 V_{Cost}\,(x_1) + w_2 V_{Sched}\,(x_2) + w_3 V_{TPerf}\,(x_3) + w_4 V_{Prgm}\,(x_4)$$

as defined by Equation 4.5. In Equation 4.8, risk score values will range between zero to one. The higher a risk event's risk score the higher its rank position in the set of identified risk events.

In Equation 4.8, values for the first term derive from a single dimensional value function that represents the probability scale in Table 4.3. Such a value function should be decided and designed by the engineering team in much the same way they are done for the impact (consequence) evaluation criteria. For discussion purposes, a linear relationship between a risk event's occurrence probability and its value function value is assumed. This is shown in Figure 4.17. Nonlinear relationships are certainly possible.

Table 4.12 presents the data for each risk event plotted in Figure 4.16. From left to right, column one is the risk event number, as labeled in Figure 4.16. Column two is the assessment of each risk event's occurrence probability. Column three is each risk event's overall impact score, computed by an application of Equation 4.5. Column four is each risk event's risk score, computed by Equation 4.8. Here, we've assumed a risk event's overall impact score is twice as important as its assessed occurrence probability. This assumption leads to *a form* of the risk score

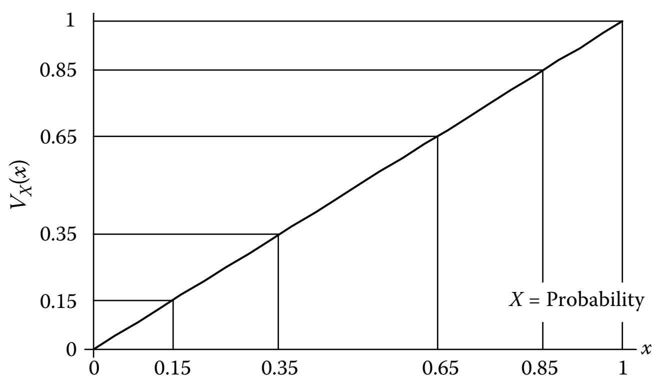


Figure 4.17: A value function for occurrence probability.

TABLE 4.12: Values and Scores for Risks in Figure 4.16

Value Scores Risk Event #	Direct Probability Assessment	Impact Score	Risk Score
1	0.95	0.477	0.635
2	0.85	0.353	0.519
3	0.65	0.867	0.795
4	0.50	0.718	0.645
5	0.93	0.688	0.769
6	0.10	0.349	0.266
7	0.95	0.194	0.446
8	0.85	0.681	0.737
9	0.35	0.695	0.580
10	0.50	0.420	0.447
11	0.70	0.516	0.578
12	0.55	0.517	0.528
13	0.10	0.515	0.376
14	0.20	0.455	0.370
15	0.95	0.432	0.605
16	0.60	0.525	0.550
17	0.75	0.382	0.505
18	0.33	0.555	0.480
19	0.25	0.254	0.252
20	0.95	0.260	0.490
21	0.45	0.248	0.315
22	0.20	0.530	0.420
23	0.35	0.475	0.434
24	0.85	0.545	0.646
25	0.80	0.481	0.587

equation given below.

$$Risk\ Score\,(E) = \frac{1}{3}Prob\,(E) + \frac{2}{3}V_{Impact}\,(E) \tag{4.9}$$

Table 4.13 presents a relative risk ranking based on the value of each risk event's risk score. Mentioned above, the higher a risk event's risk score the higher its rank position relative to other identified risks in the set. Thus, risk event 3 is in first rank position. It has the highest risk score in the set shown in Table 4.12. Risk event 5 is in second rank position. It has the second highest risk score in the set shown in Table 4.12, and so forth.

TABLE 4.13: A Relative Ranking of the Risks in Figure 4.16

Risk Ranking	
Risk Event #	**Risk Score**
3	0.795
5	0.769
8	0.737
24	0.646
4	0.645
1	0.635
15	0.605
25	0.587
9	0.580
11	0.578
16	0.550
12	0.528
2	0.519
17	0.505
20	0.490
18	0.480
10	0.447
7	0.446
23	0.434
22	0.420
13	0.376
14	0.370
21	0.315
6	0.266
19	0.252

In Table 4.13, observe the top five ranked risks are risk events 3, 5, 8, 24, and 4. This suggests the project's management should focus further scrutiny on these five risk events to further confirm they indeed merit these critical rank positions. This includes a further look at the basis of assessments behind the value function inputs chosen to characterize each risk event, as these values are used by the risk score equation to generate the above rankings.

Finally, it is best to treat any risk ranking as *indicative or suggestive* of a risk prioritization. Prioritization decisions with respect to where risk mitigation resources should be applied can be guided by this analysis but not solely directed by it. Ranking algorithms are analytical filters that serve as aids to managerial decision-making.

4.3.3 Variations on the Additive Value Model

The above illustrated the use of a simple additive value model as one way to develop a relative rank-order of risks within a set of identified risk events. This section further extends this discussion. Variations on the additive value model are presented, as well as a discussion of other rule types common and new in the engineering management community.

Variations on the Additive Value Model

Mentioned previously, risk can be considered a function of its occurrence probability and its impacts to an engineering system project. This is represented by Equation 4.10.

$$Risk = F(Probability,\ Impact) \tag{4.10}$$

What functional form is appropriate for this relationship? The answer is many. This section explores a few of these forms and offers variations on the additive value model. As we'll see, these variations are just some among many that can be designed to reflect the risk attitude of a project team or decision-maker. We will refer to these variations as formulation A, formulation B, and so forth.

Formulation A

A Weighted Linear Combination of Occurrence Probability and Impact

Here, we define the risk score of risk event E by Equation 4.11; that is,

$$Risk\ Score\,(E) = u_1 Prob\,(E) + u_2 V_{Impact}\,(E) \tag{4.11}$$

where coefficients u_1 and u_2 are non-negative weights that sum to one. The first term is a value function for the event's occurrence probability. The second term is a value function for the event's overall impact on the project; that is,

$$V_{Impact}\,(E) = w_1 V_{Cost}\,(x_1) + w_2 V_{Sched}\,(x_2) + w_3 V_{TPerf}\,(x_3) + w_4 V_{Prgm}\,(x_4)$$

as defined by Equation 4.5. In Equation 4.11, risk score values range between zero to one. The higher a risk event's risk score the higher its rank position in the set of identified risk events. Formulation A is the same as the earlier discussion on Equation 4.8.

Formulation B

A "Step-Wise" Linear Combination of Occurrence Probability and Impact

Here, we define the risk score of risk event E by Equation 4.12; that is,

$$Risk\ Score\,(E) = \begin{cases} 1 \quad \text{if } V_{Impact}\,(E) = 1 \\ u_1 Prob\,(E) + u_2 V_{Impact}\,(E) \text{ otherwise} \end{cases} \tag{4.12}$$

where the terms in Equation 4.12 are the same as defined in formulation A, but with the following change. The overall impact score of risk event E, denoted by $V_{Impact}\,(E)$, defaults to one (the maximum score) if *any* of the single dimensional value functions that constitute the terms in $V_{Impact}(E)$ reaches the value of one. If this condition arises, then the overall risk score of event E defaults to a value of one.

The philosophy behind formulation B is as follows. If a risk event E can have a maximum impact (consequence) in *any* of the specified evaluation criteria (i.e., a project's cost, schedule, technical performance, programmatic dimensions) then the overall impact score of E defaults to one; that is,

$$V_{Impact}\,(E) = 1$$

If $V_{Impact}(E) = 1$ then, according to formulation B, the risk score of event E defaults to its maximum value, which is one. Thus, if a risk event's occurrence would cause a level 5 impact on any of the project's consequence dimensions then such an effect would not be "diluted" by a weighted average rule — as in formulation A. In formulation B, risk events with a level 5 impact will always be visible to the project's management regardless of their occurrence probabilities or the importance weights of the impact (consequence) evaluation criteria.

A modification to formulation B might be as follows. The project team defines a threshold level for all values produced by the value functions used to evaluate a risk event's impact. If, for some risk event E, *each value function produces a value at or above* this threshold, then the risk score of E is equal to the maximum of that set of value function values; otherwise, the risk score of risk event E is computed by Equation 4.11. An illustration of this algorithm is shown in Figure 4.18.

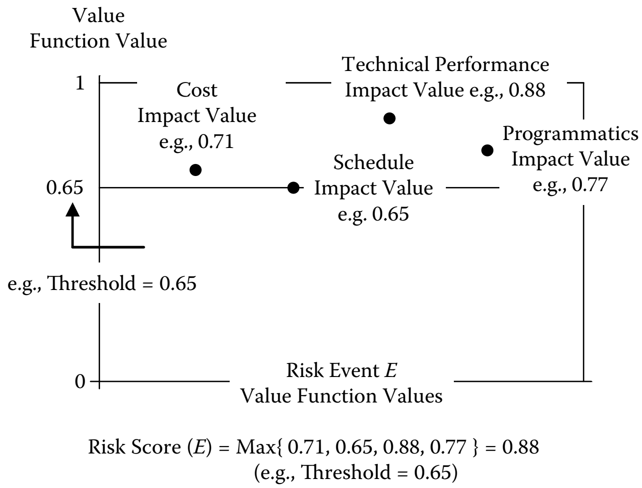


Figure 4.18: A threshold level case.

Formulation C

Maximum "Max" Average

Here, we introduce a new measure called the "max" average.* Its application as an algorithm for rank-ordering risks in a set of identified risk events will be shown. First, the definition is presented.

Definition 4.1 The max average of $x_1, x_2, x_3, \ldots, x_n$ where $0 \leq x_i \leq 1$ for all $i = 1, 2, 3, \ldots, n$ is

$$Max\ Ave = \lambda m + (1 - \lambda) Average\{x_1, x_2, x_3, \ldots, x_n\} \tag{4.13}$$

where $m = Max\{x_1, x_2, x_3, \ldots, x_n\}$ and λ is a weighting function given by Equation 4.14.

The weighting function λ in Equation 4.14 is shown in Figure 4.19. It is a form of a function known as the sigmoid function. This is one of many possible forms

*The max average was created by Dr. Bruce W. Lamar (MITRE, 2005) and published by The MITRE Corporation in the paper *Min-Additive Utility Functions*, MP080070-1, April 2008, © 2008, All Rights Reserved.

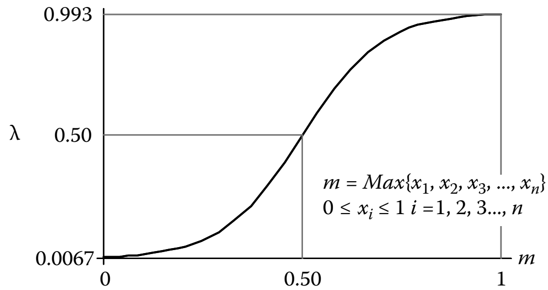


Figure 4.19: A max average weighting function λ.

of a weight function. Another form for λ is shown in Figure 4.53 as part of that section's discussion.

$$0 < \lambda = 1 - \frac{1}{1 + e^{10(m-1/2)}} = \frac{e^{10(m-1/2)}}{1 + e^{10(m-1/2)}} < 1 \tag{4.14}$$

In Equation 4.14, if $m > 0.50$ then the max average of $x_1, x_2, x_3, \ldots, x_n$ is weighted toward the maximum of $\{x_1, x_2, x_3, \ldots, x_n\}$. If $m < 0.50$, then the max average of $x_1, x_2, x_3, \ldots, x_n$ is weighted toward the arithmetic average of $x_1, x_2, x_3, \ldots, x_n$. If $m = 0.50$, then the max average of $x_1, x_2, x_3, \ldots, x_n$ is weighted equally ($\lambda = 0.50$) between the maximum of $x_1, x_2, x_3, \ldots, x_n$ and the arithmetic average of $x_1, x_2, x_3, \ldots, x_n$. Next, the max average will be applied to the problem of rank-ordering risks in a set of identified events.

One way to apply the max average rule is to define $V_{Impact}(E)$ as follows:

$$V_{Impact}(E) = \lambda Max\{v_{Cost}, v_{Sched}, v_{TPerf}, v_{Prgm}\} + (1 - \lambda) Weighted\ Average\{w_1 v_{Cost}, w_2 v_{Sched}, w_3 v_{TPerf}, w_4 v_{Prgm}\} \tag{4.15}$$

where $v_{Cost} = V_{Cost}(x_1)$, $v_{Sched} = V_{Sched}(x_2)$, $v_{TPerf} = V_{TPerf}(x_3)$, and $v_{Prgm} = V_{Prgm}(x_4)$ are defined by Equation 4.5. Here, w_i (for $i = 1, \ldots, 4$) are non-negative importance weights with values that range between zero and one and where $w_1 + w_2 + w_3 + w_4 = 1$.

The second term in Equation 4.15 is computed as follows:

$$Weighted\ Average\{w_1 v_{Cost}, w_2 v_{Sched}, w_3 v_{TPerf}, w_4 v_{Prgm}\} = w_1 v_{Cost} + w_2 v_{Sched} + w_3 v_{TPerf} + w_4 v_{Prgm} \tag{4.16}$$

Using Equation 4.15 for $V_{Impact}(E)$, formulation A could then be used to compute the risk score of event E; that is,

$$Risk\ Score\,(E) = u_1 Prob\,(E) + u_2 V_{Impact}(E) \tag{4.17}$$

Example 4.1 From Case Discussion 4.1, compute the risk score of risk event A using Equation 4.17 and the max average rule given by Equation 4.15. Here, assume the overall impact score of risk event A is twice as important as its assessed occurrence probability.

Solution From Case Discussion 4.1, Equation 4.8, and the assumption that risk event A's overall impact score is twice as important as its assessed occurrence probability we have the following:

$$Risk\ Score(A) = \frac{1}{3} Prob\,(A) + \frac{2}{3} V_{Impact}\,(A) \tag{4.18}$$

Next, we'll use the max average rule to compute $V_{Impact}(A)$. From Case Discussion 4.1, Table 4.11, and Equation 4.15 we have the following:

$$v_{Cost} = 0.842, \quad v_{Sched} = 0.604, \quad v_{TPerf} = 0.60, \quad \text{and} \quad v_{Prgm} = 0.79$$

From the above, it follows that

$$m = Max\{v_{Cost}, v_{Sched}, v_{TPerf}, v_{Prgm}\} = Max\{0.842, 0.604, 0.60, 0.79\} = 0.842$$

Next, we compute

$$\begin{aligned} &Weighted\ Average\{w_1 v_{Cost}, w_2 v_{Sched}, w_3 v_{TPerf}, w_4 v_{Prgm}\} \\ &\quad = w_1 v_{Cost} + w_2 v_{Sched} + w_3 v_{TPerf} + w_4 v_{Prgm} \end{aligned} \tag{4.19}$$

where, from Case Discussion 4.1, the weights were 1/4, 1/8, 1/2, 1/8. Hence

$$\begin{aligned} &Weighted\ Average\left\{\frac{1}{4} v_{Cost}, \frac{1}{8} v_{Sched}, \frac{1}{2} v_{TPerf}, \frac{1}{8} v_{Prgm}\right\} \\ &\quad = \frac{1}{4} v_{Cost} + \frac{1}{8} v_{Sched} + \frac{1}{2} v_{TPerf} + \frac{1}{8} v_{Prgm} = 0.685 \end{aligned} \tag{4.20}$$

From Equation 4.15 we have

$$V_{Impact}(A) = \lambda(0.842) + (1 - \lambda)(0.685)$$

where

$$\lambda = 1 - \frac{1}{1 + e^{10(m-1/2)}} = 1 - \frac{1}{1 + e^{10(0.842-1/2)}} = 0.968$$

It follows that

$$V_{Impact}(A) = 0.968(0.842) + (1 - 0.968)(0.685) = 0.837$$

from which

$$Risk\ Score(A) = \frac{1}{3}Prob(A) + \frac{2}{3}V_{Impact}(A) \tag{4.21}$$

$$Risk\ Score(A) = \frac{1}{3}(0.95) + \frac{2}{3}(0.837) = 0.875 \tag{4.22}$$

Figure 4.20 shows a plot of risk event A. Two points are shown. The left-most point is a plot of risk event A if the event's overall impact score is computed by a strict weighted average rule (refer to Equation 4.7). The right-most point is a plot of risk event A if the event's overall impact score is computed using the max average rule (refer to the use of Equation 4.15 in example 4.1). Why is there such a difference? When do you choose one rule over another?

The answer to the first question can be seen in the technical differences between Equations 4.7 and 4.15. In Equation 4.7, a risk event's overall impact score is a weighted average of its individual impact scores (i.e., the value function values) across the evaluation criteria. Equation 4.15 uses this same weighted average in

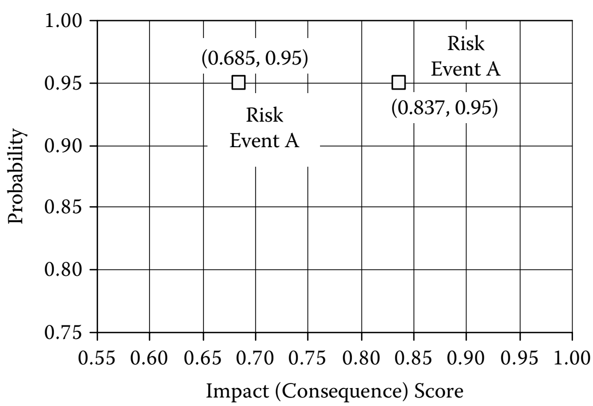


Figure 4.20: Example 4-1: a plot of risk event A: two scoring rules.

its second term; however, the first term of Equation 4.15 takes the maximum value of the risk event's individual impact scores across the evaluation criteria.

In Example 4.1, the maximum value received almost 97% of the importance weight while the weighted average received approximately 3% of the importance weight. In this case, the maximum of the individual impact scores dominated the overall impact score.

The answer to the second question is driven by the risk attitude or impact sensitivity of the project team. The max average rule "values" the maximum *m* more than the weighted average when *m* is greater than 0.50. The max average rule "values" the average more than the maximum when *m* is less than 0.50. The max average rule "equally values" the maximum *m* and the average when *m* is equal to 0.50, each receiving a weight equal to one-half. These characteristics can be seen by a close examination of Figure 4.19, a graph of Equation 4.14.

Formulation D

Product Rule

The "product rule" is a popular formulation in the risk management community. The product rule defines the risk score of risk event E as the product of the event's occurrence probability and its impact (consequence). A traditional form of the product rule is given by Equation 4.23.

$$Risk\ Score(E) = Prob(E) \cdot V_{Impact}(E) \tag{4.23}$$

Here, the first term is a value function for the event's occurrence probability.* The second term is a value function for the event's overall impact on the project, as defined by Equation 4.5.

From a statistical perspective, the product rule generates a measure known as an expected value [5]. In this context, $Risk\ Score(E)$ could be interpreted as the expected impact of risk event E.

The rule given by Equation 4.23 produces meaningful results *only* when its terms are *defined along cardinal scales*. For reasons discussed in Chapter 3 and in section 4.3.1, Equation 4.23 should not be used when either *Prob(E)* or $V_{Impact}(E)$ are ordinal valued.

*Refer to the discussion on Equation 4.8 and Figure 4.17 for a further discussion of this value function.

A shortcoming with the product rule is the inability to weight the importance of $Prob(E)$ versus $V_{Impact}(E)$. However, importance weights for the evaluation criteria that constitute $V_{Impact}(E)$ can be weighted. Here, Equation 4.23 could be written as follows:*

$$\begin{aligned} Risk\ Score(E) &= Prob(E) \cdot V_{Impact}(E) \\ &= Prob(E) \cdot (w_1 v_{Cost} + w_2 v_{Sched} + w_3 v_{TPerf} + w_4 v_{Prgm}) \end{aligned} \tag{4.24}$$

where $v_{Cost} = V_{Cost}(x_1)$, $v_{Sched} = V_{Sched}(x_2)$, $v_{TPerf} = V_{TPerf}(x_3)$, and $v_{Prgm} = V_{Prgm}(x_4)$ are defined by Equation 4.5. Here, w_i (for $i = 1, \ldots, 4$) are non-negative importance weights with values that range between zero and one and where $w_1 + w_2 + w_3 + w_4 = 1$.

Related to the above, the product rule is structured in a way where an event's occurrence probability can significantly discount its overall potential impact (consequence) to a project. The preceding formulations had an event's occurrence probability as additive to impact (consequence) and not multiplicative. In some circumstances, Equation 4.23 can give a false sense of comfort. For example, a high-impact risk event can appear less threatening to a project if its occurrence probability is determined to be low. Allowing occurrence probability this much "influence" on an event's risk score may not be a good idea, since a subjective assessment of an event's occurrence probability can be off by a wide margin and is often made without strong defensible arguments. In practice, projects tend to be more sensitive to a risk event's impact than its probability. As such, a high-consequence risk event should never lose visibility or be mistakenly downplayed, because its occurrence probability is merely considered low. Program managers and decision-makers should always be presented with both values.

There are other related issues associated with this rule. The product rule can produce risk scores that are the same, or close in value, for two very different risk events. Seen in Figure 4.21, a risk event with a low-impact and a high-occurrence probability can produce the same risk score as an event with a high-impact and a low-occurrence probability. In these circumstances, the individual values for $Prob(E)$ and $V_{Impact}(E)$ should be flagged so program managers and decision-makers can assess where tradeoffs may best be made.

*Formulas for $V_{Impact}(E)$ developed in formulations B and C could also be used in the "product-rule" formulation for risk score.

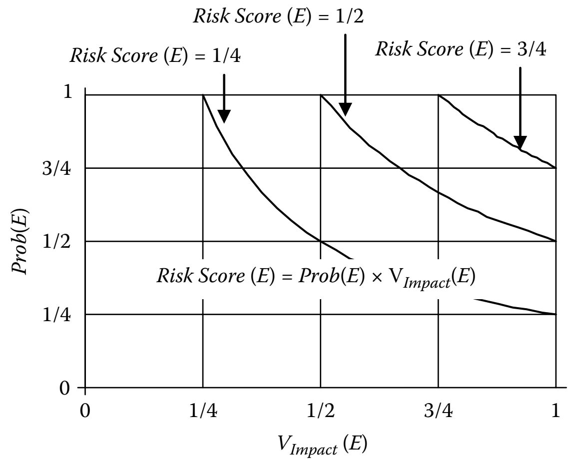


Figure 4.21: Curves of constant risk score.

Example 4.2 From Case Discussion 4.1, compute the risk score of risk event A using the product rule with

a) $V_{Impact}(E)$ derived from Equation 4.6 and Equation 4.7

b) $V_{Impact}(E)$ derived from Equation 4.15 and illustrated in example 4.1

Solution From Case Discussion 4.1, we have the following:

$B = \{$The new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released$\}$

$A = \{$Inadequate synchronization of the communication system's new database with the existing subsystem databases$\}$

It was also assessed that the occurrence probability of risk event A was

$$0 < P(A|B) = \alpha = 0.95 < 1$$

Using the product rule, we would write:

$$Risk\ Score(A) = 0.95 \cdot V_{Impact}(E) \tag{4.25}$$

(a) From Equation 4.6, we determined that

$$V_{Impact}(A) = \frac{1}{4}V_{Cost}(x_1) + \frac{1}{8}V_{Sched}(x_2) + \frac{1}{2}V_{TPerf}(x_3) + \frac{1}{8}V_{Prgm}(x_4)$$

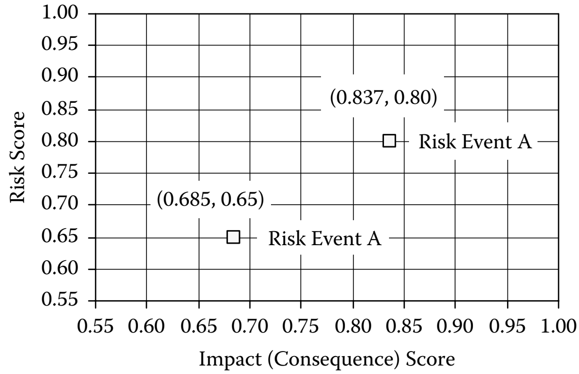


Figure 4.22: A plot of risk score versus impact: Example 4.2.

which, from Case Discussion 4.1 (Equation 4.7) is

$$V_{Impact}(A) = \frac{1}{4}(0.842) + \frac{1}{8}(0.604) + \frac{1}{2}(0.60) + \frac{1}{8}(0.79) = 0.685 \qquad (4.26)$$

It follows that

$$Risk\ Score(A) = 0.95 \cdot (0.685) = 0.65 \qquad (4.27)$$

(b) From Equation 4.15 and Example 4.1 we have

$$V_{Impact}(A) = 0.837 \qquad (4.28)$$

It follows that

$$Risk\ Score(A) = 0.95 \cdot (0.837) = 0.80 \qquad (4.29)$$

Figure 4.22 presents a plot of these two values for risk score, as a function of risk event A's impact.

$$Risk\ Score(A) = Prob(A) \cdot V_{Impact}(A)$$

4.3.4 Incorporating Uncertainty

. . . The only certainty is uncertainty.

Pliny the Elder (Gaius Plinius Secundus)

This section illustrates an application of the power-additive utility function as a way to capture uncertainty, and the risk attitude of the decision-maker, in the analysis of one or more risk events. The power-additive utility function was introduced in Chapter 3. It is a function that takes values from a multiattribute value function and maps them into a corresponding set of utilities in accordance with the risk attitude of the decision-maker. The power-additive utility function covers a wide span of possible risk attitudes, as shown in Figure 3.24. In this section, we limit our focus to the situation where the utilities are monotonically increasing.

Concept Review (from Chapter 3)

Recall the following from Chapter 3, Definition 3.12. If utilities are *monotonically increasing* over the values of the additive value function $V_Y(y)$, then the power-additive utility function is given by

$$U(v) = \begin{cases} K(1 - e^{-(V_Y(y)/\rho_m)}) & \text{if } \rho_m \neq \infty \\ V_Y(y) & \text{if } \rho_m = \infty \end{cases} \tag{4.30}$$

where $K = 1/(1 - e^{-1/\rho_m})$, $v = V_Y(y) = \sum_{i=1}^{n} w_i V_{X_i}(x_i)$, ρ_m is the multiattribute risk tolerance, and $V_Y(y)$ is the additive value function given in Definition 3.6.

From Chapter 3, recall that expected utilities provide measures with which to rank uncertain alternatives, from most-to least-preferred. In a risk event context, we will compute the expected utilities of their values as a way to rank them, from most-to least-critical, when uncertainties are present in the characteristics of these events.

From Theorem 3.4, if utilities are *monotonically increasing* over the attributes of the additive value function $V_Y(y)$, then the expected utility $E(U(v))$ is given below.

$$E(U(v)) = \begin{cases} K(1 - E(e^{-(V_Y(y)/\rho_m)})) & \text{if } \rho_m \neq \infty \\ E(V_Y(y)) & \text{if } \rho_m = \infty \end{cases} \tag{4.31}$$

For the case where $\rho \neq \infty$, the term $E(e^{-(V_Y(y)/\rho_m)})$ can be written as follows:

$$E(e^{-(V_Y(y)/\rho_m)}) = E(e^{-(w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) \ldots w_n V_{X_n}(x_n))/\rho_m})$$
$$E(e^{-(V_Y(y)/\rho_m)}) = E(e^{-(w_1 V_{X_1}(x_1))/\rho_m}) E(e^{-(w_2 V_{X_2}(x_2))/\rho_m}) \ldots E(e^{-(w_n V_{X_n}(x_n))/\rho_m})$$

where the X_i's are independent random variables and where

$$E(e^{-(w_i V_{X_i}(x_i))/\rho_m}) = \begin{cases} \sum_{x_i} p_{X_i}(x_i)\, e^{-(w_i V_{X_i}(x_i))/\rho_m} & \text{if } X_i \text{ is discrete} \\ \int_{-\infty}^{\infty} e^{-(w_i V_{X_i}(x_i))/\rho_m} f_{X_i}(x_i)\, dx_i & \text{if } X_i \text{ is continuous} \end{cases} \tag{4.32}$$

In the above, $p_{X_i}(x_i)$ is the probability the uncertain outcome X_i takes the score x_i if X_i is a discrete random variable and $f_{X_i}(x_i)$ is the probability density function of X_i if X_i is a continuous random variable.

Case Discussion 4.1a: An Application Illustration

The following is an extension of Case Discussion 4.1. It shows how to compute the expected utility of the value of a risk event, for a given risk attitude, in the presence of uncertainty in the parameters that characterize the event.

Although the approach in Case Discussion 4.1a is illustrated for one risk event, it can be applied to each risk event in a set of identified events; that is, the expected utility of the value of each risk can be computed as (1) a function of the uncertainties that characterize each event and (2) the risk attitude of the program manager or decision-maker. From this, a most-to least-critical ranking of each risk event can be determined from these expected utility measures. Higher criticality events have higher expected utilities, and so forth.

Computing the Expected Utility

From the value functions given in Table 4.14, we can write the following additive value function.

$$V_Y(y) = u_1 V_Z(z) + u_2(w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + w_3 V_{X_3}(x_3) + w_4 V_{X_4}(x_4))$$

Here, we assume the conditions defined in Chapter 3 for an additive value function hold. Observe the preceding equation is an equivalent formulation to Equation 4.8, called *Risk Score*. Here, the first term is the value function for the occurrence probability of the risk event (refer to Figure 4.17). The remaining terms are equivalent to the terms that make up the value function for the risk event's overall impact, defined by Equation 4.5; that is, X_1 denotes the criterion Cost Impact; X_2 denotes the criterion Schedule Impact, X_3 denotes the criterion Technical Performance Impact, and X_4 denotes the criterion Programmatic Impact.

Applying the weights from Case Discussion 4.1, and Equation 4.9, we have the following:

$$V_Y(y) = \frac{1}{3} V_Z(z) + \frac{2}{3}\left(\frac{1}{4} V_{X_1}(x_1) + \frac{1}{8} V_{X_2}(x_2) + \frac{1}{2} V_{X_3}(x_3) + \frac{1}{8} V_{X_4}(x_4)\right)$$

TABLE 4.14: Uncertainty Assessments and Scores from Case Discussion 4.1

Risk Event A	Basis of Assessment
Uncertainty Assessments	Risk Statement *Inadequate synchronization of the communication system's new database with the existing subsystem databases, because the new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released.*
Criterion Z Occurrence Probability	This risk event is assessed to have between an 85% and a 95% chance of occurrence. Value Function: From Figure 4.17, $V_Z(z) = z$ *Note: Suppose the uncertainty is uniformly distributed.*
Criterion X_1 Cost Impact	This risk event, if it occurs, is estimated by the engineering team to cause a 10% to 15% increase in the project's current budget. The estimate is based on a careful assessment of the ripple effects across the project's cost categories for interoperability fixes to the databases, the supporting software, and the extent that retesting is needed. Value Function: From Equation 4.2, $V_{X_1}(x_1) = 1.096(1 - e^{-x_1/8.2})$ *Note: Suppose the uncertainty is uniformly distributed.*
Criterion X_2 Schedule Impact	This risk event, if it occurs, is estimated by the engineering team to cause a 3-month to 6-month increase in the project's current schedule. The estimate is based on a careful assessment of the ripple effects across the project's integrated master schedule for interoperability-related fixes to the databases, the supporting software, and the extent that retesting is needed. Value Function: From Equation 4.3, $V_{X_2}(x_2) = 1.018(1 - e^{-x_2/4.44})$ *Note: Suppose the uncertainty is uniformly distributed.*

TABLE 4.14: Uncertainty Assessments and Scores from Case Discussion 4.1 (*Continued*)

Risk Event A	**Basis of Assessment**
Criterion X_3 Technical Performance Impact	This risk event, if it occurs, is assessed by the engineering team as one that will impact the system's operational capabilities to the extent that technical performance is marginally below minimum acceptable levels, depending on the location and extent of interoperability shortfalls. Given this, suppose the engineering team assessed a 75% chance this risk event would have a Level 4 technical performance impact and a 25% chance it would have a Level 3 impact. Value Function: From Figure 4.12, $V_{X_3}(4) = 9/15$ and $V_{X_3}(3) = 5/15$
Criterion X_4 Programmatic Impact	This risk event, if it occurs, is assessed by the engineering team as one that will impact programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (e.g., various specifications or activities) is marginally below minimum acceptable levels. Given this, suppose the engineering team assessed a 60% chance this risk event would have a Level 4 technical performance impact and a 40% chance it would have a Level 3 impact. Value Function: From Figure 4.12, $V_{X_4}(4) = 15/19$ and $V_{X_4}(3) = 9/19$

which is equal to

$$V_Y(y) = \frac{1}{3}V_Z(z) + \frac{1}{6}V_{X_1}(x_1) + \frac{1}{12}V_{X_2}(x_2) + \frac{1}{3}V_{X_3}(x_3) + \frac{1}{12}V_{X_4}(x_4) \quad (4.33)$$

Suppose decision-makers reviewed the graphs in Figure 3.24 (Chapter 3) and assessed their multiattribute risk tolerance as represented by the curve with $\rho_m = 1$. So, their risk preference structure reflects a monotonically increasing, slightly risk-averse attitude over increasing values of the value function in Equation 4.33. From this, and the information in Table 4.14, we can now compute the expected utility of the value of the risk event defined in Case Discussion 4.1.

From Theorem 3.4 we have

$$E(U(v)) = K(1 - E(e^{-(V_Y(y)/\rho_m)})) \tag{4.34}$$

where $K = 1/(1 - e^{-1/\rho_m})$ and $v = V_Y(y)$ as given by Equation 4.33. It follows that with $\rho_m = 1$ Equation 4.34 becomes

$$E(U(v)) = 1.582(1 - E(e^{-V_Y(y)})) \tag{4.35}$$

In Equation 4.35, we have

$$v = V_Y(y) \tag{4.36}$$

$$= \frac{1}{3}V_Z(z) + \frac{1}{6}V_{X_1}(x_1) + \frac{1}{12}V_{X_2}(x_2) + \frac{1}{3}V_{X_3}(x_3) + \frac{1}{12}V_{X_4}(x_4) \tag{4.37}$$

where

$$V_Z(z) = z \tag{4.38}$$

$$V_{X_1}(x_1) = 1.096(1 - e^{-x_1/8.2}) \tag{4.39}$$

$$V_{X_2}(x_2) = 1.018(1 - e^{-x_2/4.44}) \tag{4.40}$$

and $V_{X_3}(x_3)$ and $V_{X_4}(x_4)$ are given by the value functions in Figure 4.12. Next, we will look at computing the term $E(e^{-V_Y(y)})$ in Equation 4.35. Here,

$$E(e^{-V_Y(y)}) = E(e^{-(\frac{1}{3}V_Z(z)+\frac{1}{6}V_{X_1}(x_1)+\frac{1}{12}V_{X_2}(x_2)+\frac{1}{3}V_{X_3}(x_3)+\frac{1}{12}V_{X_4}(x_4))})$$

If we assume Z, X_1, X_2, X_3, and X_4 are independent random variables* then

$$E(e^{-V_Y(y)}) = E(e^{-\frac{1}{3}V_Z(z)})E(e^{-\frac{1}{6}V_{X_1}(x_1)})E(e^{-\frac{1}{12}V_{X_2}(x_2)})E(e^{-\frac{1}{3}V_{X_3}(x_3)})E(e^{-\frac{1}{12}V_{X_4}(x_4)}) \tag{4.41}$$

*This assumption will be discussed further at the end of this section.

where

$$E(e^{-\frac{1}{3}V_Z(z)}) = \int_{-\infty}^{\infty} e^{-\frac{1}{3}V_Z(z)} f_Z(z)\,dz \tag{4.42}$$

$$E(e^{-\frac{1}{6}V_{X_1}(x_1)}) = \int_{-\infty}^{\infty} e^{-\frac{1}{6}V_{X_1}(x_1)} f_{X_1}(x_1)\,dx_1 \tag{4.43}$$

$$E(e^{-\frac{1}{12}V_{X_2}(x_2)}) = \int_{-\infty}^{\infty} e^{-\frac{1}{12}V_{X_2}(x_2)} f_{X_2}(x_2)\,dx_2 \tag{4.44}$$

$$E(e^{-\frac{1}{3}V_{X_3}(x_3)}) = \sum_{x_3} p_{X_3}(x_3)\, e^{-\frac{1}{3}V_{X_3}(x_3)} \tag{4.45}$$

$$E(e^{-\frac{1}{12}V_{X_4}(x_4)}) = \sum_{x_4} p_{X_4}(x_4)\, e^{-\frac{1}{12}V_{X_4}(x_4)} \tag{4.46}$$

and $f_Z(z)$, $f_{X_i}(x_i)$, and $p_{X_i}(x_i)$ are probability distributions for Z and X_i which, in this case discussion, are stated in Table 4.14. From Table 4.14, the above can be computed, as given below.

$$E(e^{-\frac{1}{3}V_Z(z)}) = \int_{0.85}^{0.95} e^{-\frac{1}{3}z} \frac{1}{0.95-0.85}\,dz = 0.740853 \tag{4.47}$$

$$E(e^{-\frac{1}{6}V_{X_1}(x_1)}) = \int_{10}^{15} e^{-\frac{1}{6}(1.096(1-e^{-x_1/8.2}))} \frac{1}{15-10}\,dx_1 = 0.867408 \tag{4.48}$$

$$E(e^{-\frac{1}{12}V_{X_2}(x_2)}) = \int_{3}^{6} e^{-\frac{1}{12}(1.018(1-e^{-x_2/4.44}))} \frac{1}{6-3}\,dx_2 = 0.947966 \tag{4.49}$$

$$\begin{aligned} E(e^{-\frac{1}{3}V_{X_3}(x_3)}) &= \sum_{x_3} p_{X_3}(x_3)\, e^{-\frac{1}{3}V_{X_3}(x_3)} \\ &= 0.75\,e^{-\frac{1}{3}\frac{9}{15}} + 0.25\,e^{-\frac{1}{3}\frac{5}{15}} = 0.837758 \end{aligned} \tag{4.50}$$

$$\begin{aligned} E(e^{-\frac{1}{12}V_{X_4}(x_4)}) &= \sum_{x_4} p_{X_4}(x_4)\, e^{-\frac{1}{12}V_{X_4}(x_4)} \\ &= 0.60\,e^{-\frac{1}{12}\frac{15}{19}} + 0.40\,e^{-\frac{1}{12}\frac{9}{19}} = 0.946315 \end{aligned} \tag{4.51}$$

Entering these values into Equation 4.41 we have

$$E(e^{-V_Y(y)}) = (0.740853)(0.867408)(0.947966)(0.837758)(0.946315) = 0.48295 \tag{4.52}$$

Substituting this value for $E(e^{-V_Y(y)})$ into Equation 4.35 we have

$$E(U(v)) = 1.582(1 - 0.48295) = 0.817961 \tag{4.53}$$

Computing the Expected Value

Next, we proceed to compute the expected value of the risk event's value. Here, we need to determine $E(v)$ where,

$$\begin{aligned} E(v) &= E(V_Y(y)) \\ &= E\left(\frac{1}{3}V_Z(z) + \frac{1}{6}V_{X_1}(x_1) + \frac{1}{12}V_{X_2}(x_2) + \frac{1}{3}V_{X_3}(x_3) + \frac{1}{12}V_{X_4}(x_4)\right) \\ &= \frac{1}{3}E(V_Z(z)) + \frac{1}{6}E(V_{X_1}(x_1)) + \frac{1}{12}E(V_{X_2}(x_2)) \\ &\qquad + \frac{1}{3}E(V_{X_3}(x_3)) + \frac{1}{12}E(V_{X_4}(x_4)) \end{aligned}$$

The terms in the above expression are determined, in this case, as follows:

$$E(V_Z(z)) = \int_{-\infty}^{\infty} V_Z(z)\, f_Z(z) dz = \int_{0.85}^{0.95} z\, \frac{1}{0.95 - 0.85} dz = 0.90$$

$$E(V_{X_1}(x_1)) = \int_{-\infty}^{\infty} V_{X_1}(x_1)\, f_{X_1}(x_1) dx_1 = \int_{10}^{15} 1.096(1 - e^{-x_1/8.2})\, \frac{1}{15 - 10} dx_1 = 0.853627$$

$$E(V_{X_2}(x_2)) = \int_{-\infty}^{\infty} V_{X_2}(x_2)\, f_{X_2}(x_2) dx_2 = \int_{3}^{6} 1.018(1 - e^{-x_2/4.44})\, \frac{1}{6 - 3} dx_2 = 0.641457$$

$$E(V_{X_3}(x_3)) = \sum_{x_3} p_{X_3}(x_3)V_{X_3}(x_3) = 0.75\left(\frac{9}{15}\right) + 0.25\left(\frac{5}{15}\right)$$
$$= 0.533333$$
$$E(V_{X_4}(x_4)) = \sum_{x_4} p_{X_4}(x_4)V_{X_4}(x_4) = 0.60\left(\frac{15}{19}\right) + 0.40\left(\frac{9}{19}\right) = 0.663158$$

Substituting these values into the above expression for $E(v)$ we have

$$E(v) = \frac{1}{3}(0.90) + \frac{1}{6}(0.853627) + \frac{1}{12}(0.641457)$$
$$+ \frac{1}{3}(0.533333) + \frac{1}{12}(0.663158) = 0.728767 \qquad (4.54)$$

Some Observations

This case discussion looked at how to incorporate uncertainty in the parameters that characterized one risk event. Two measures were computed — the expected utility $E(U(v))$ of the value of the risk event and the expected value of its value, denoted by $E(v)$. Here, the value of the risk event was measured by the value function we called *Risk Score*, as given by Equation 4.8.

In the preceding case discussion, the multiattribute risk tolerance was set equal to one ($\rho_m = 1$). This means the program manager or decision-maker had a slight degree of risk averseness over the values of the value function. A graph of this utility function is presented in Figure 4.23.

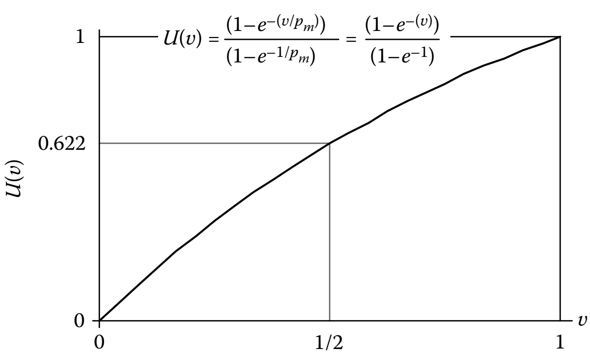


Figure 4.23: Utility function for Case Discussion 4.1a.

From the above, we determined that $E(U(v)) = 0.817961$ and $E(v) = 0.728767$. Recall from Chapter 3 that, in the case of risk averseness, the utility of the expected value should be larger than the expected utility. This can be seen here as well. In this case, we have

$$U(E(v)) = U(0.728767) = 0.818667 > E(U(v)) = 0.817961$$

Next, it might be asked: *What is the value of v associated with the expected utility*? This would be a measure known as the *certainty equivalent value* of the risk event, in this case (refer to Chapter 3). To determine the certainty equivalent value we solve the expression below for v_{CE}.

$$E(U(v)) = 0.817961 = \frac{(1 - e^{-(v_{CE})})}{(1 - e^{-1})} = U(v_{CE})$$

With a little algebra, it can be shown that $v_{CE} = 0.727842$. So, this is the value of the value function that produces the expected utility of the value of the risk event. Notice v_{CE} is slightly less than $E(v)$, as expected. A graph of these observations is shown in Figure 4.24. Figure 4.24 "narrows-in" on the utility function in Figure 4.23 for the region $0.725 \leq v \leq 0.73$. Notice how linear the function looks in this very tight interval.

Finally, let's take another look at Case Discussion 4.1 as originally presented without considering uncertainty or risk preferences (i.e., risk attitudes). Suppose we use formulation A to measure the risk score of risk event A, with weights

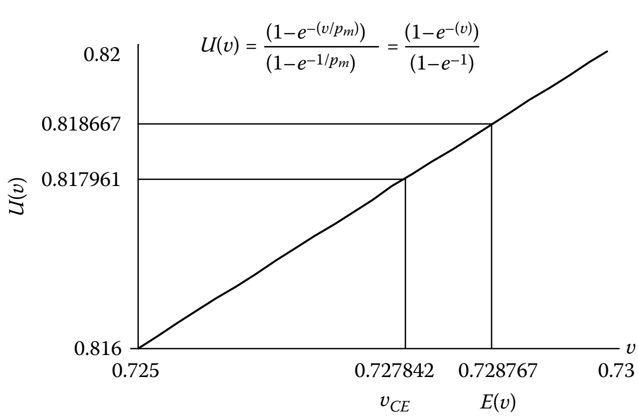


Figure 4.24: Some utility function values for case discussion 4.1a.

shown in Equation 4.55.

$$Risk\ Score\ (A) = \frac{1}{3}Prob(A) + \frac{2}{3}V_{Impact}(A) \tag{4.55}$$

From the results of Case Discussion 4.1, we can compute the following:

$$Risk\ Score\ (A) = \frac{1}{3}(0.95) + \frac{2}{3}(0.685) = 0.77333 \tag{4.56}$$

Notice how this value is greater than $E(v)$. The reason is that $E(v)$ incorporates uncertainties in the values for the parameters that characterize the risk event. These uncertainties were given in Table 4.14, where there were enough "lower possible" scores, in this case, to drive $E(v)$ below the value of risk score, as computed by Equation 4.56.

Some Words on Probabilistic Independence

In Case Discussion 4.1a, a key assumption was that Z, X_1, X_2, X_3, and X_4 were independent random variables. There is a practical reason for making this assumption. Without it, Equation 4.41 could not be written as shown.

However, there are occasions when random variables, such as these, are not independent. For example, the uncertainties in the levels (or scores) for Cost (X_1) and Schedule (X_2) can sometimes vary together, not independently. When this occurs, expected utility computations can be in error, if independence assumptions were wrongly made. The error worsens with greater degrees of risk averseness. The error lessens with fewer degrees of risk averseness and lessens substantially when the utility function approaches a straight line, otherwise known as the risk neutral condition. The error goes to zero when the utility function is linear since, in this situation, the expected utility is exactly equal to the expected value. From probability theory [5], it can be shown that expected value computations are not subject to independence or dependence considerations.

When probabilistic independence can't be assumed or demonstrated, one approach is to redefine the dependent random variables in a way that they will exhibit independence from the rest of the random variables in the set. For example, in Case Discussion 4.1a, if Cost (X_1) and Schedule (X_2) were not independent then one could do the following. Redefine the value function for Cost such that it captures the joint interactions between cost and schedule. In this way, the new value function would be purposefully designed to incorporate schedule effects. Hence, Schedule (X_2) would no longer have its own value function or be included in the list. Independence between Z, X_1, X_3, and X_4 might then be reasonably assumed.

4.4 Risk Management and Progress Monitoring

This section presents approaches for managing and monitoring the progress of risk handling plans, tasks, and actions. Measures are developed that enable management to measure progress over time, as well as identify mitigation activities most responsible for impeding progress. The section begins with a review of the classical four risk handling strategies.

4.4.1 Risk Handling Approaches

There are a variety of ways a project's management handles risk. In general, these strategies can be categorized by one of the following actions. These are *Risk Avoidance, Risk Control*, *Risk Acceptance*, or *Risk Transfer*. The following defines these approaches in accordance with excerpts from reference 1.

Risk Avoidance: "Risk Avoidance involves a change in the concept, requirements, specifications, and/or practices that reduce risk to an acceptable level. A risk avoidance strategy eliminates the sources of high or possibly medium risks and replaces them with a lower risk solution. Such a solution should be supported by a corresponding cost-benefit analysis. Generally, this strategy may be conducted in parallel with up-front capability planning or requirements analyses and supported by cost-tradeoff studies." [1]

Risk Assumption: "Risk assumption is an acknowledgment of the existence of a particular risk situation and a conscious decision to accept the associated level of risk without engaging in special efforts to control it. However, a general cost and schedule reserve may be set aside to deal with any problems that may occur as a result of various risk assumption decisions. This strategy recognizes that not all identified program risks warrant special handling; as such, it is most suited for those situations that have been classified as low risk." [1]

Risk Transfer: "This strategy is one that reallocates risk from one part of the project to another or redistributing risks between the organization (e.g., the government) acquiring the system and the system's prime contractor. Risk transfer is a form of risk sharing. It should not be viewed as risk abrogation. An example is the transfer of a function from hardware implementation to software implementation. The effectiveness of risk transfer depends on the use of successful system engineering techniques, such as modular design and functional partitioning techniques." [1]

Risk Control: "Risk control actively engages strategies to reduce or mitigate risk. It monitors and manages risk in a manner that reduces its occurrence probability and/or consequences on the project. Risk control is a widely exercised handling strategy by a project's management. Because of this, various approaches to monitoring the progress of mitigation strategies have been developed [1, 2, 3]. The following presents one of the newer approaches." [1]

4.4.2 Monitoring Progress — A Performance Index Measure

This section presents an approach for monitoring the progress of risk mitigation plans that are composed of individual activities or tasks. An index is developed that enables management to measure progress and view trends that may reveal where corrective actions are most needed.

There is little in the way of analytical formalisms for monitoring the time-history progress of activities or tasks that constitute a risk event's risk mitigation plan. In practice, activities are often simply color coded to indicate progress, with little explanation or consistency for their basis. This is illustrated in the figure below, which shows a traditional waterfall model of risk mitigation progress.

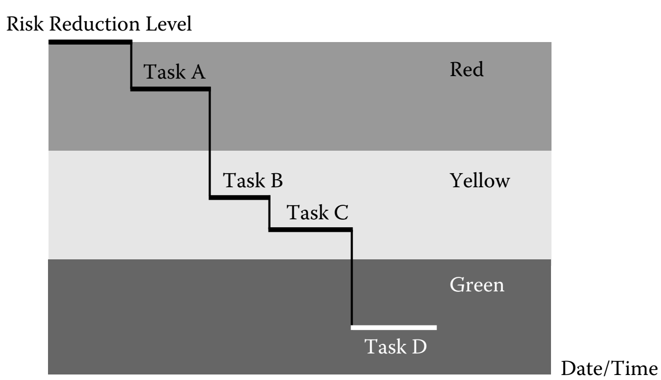


Figure 4.25: A traditional waterfall model view.

Here, a series of tasks are undertaken to reduce risk. Each task is monitored in terms of its duration and the extent that, when completed, it has reduced risk. However, the magnitude that any one task reduces risk is highly subjective. Assessments are typically made without being supported by objective measures. They are, essentially, "guesstimates." The approach described in this section lessens this guesswork. It provides a visible and traceable basis for assessing the extent that risk mitigation progress is truly being made.

Measuring the Progress of Risk Mitigation Plans

Instead of the above, suppose each risk's risk mitigation plan is treated as a portfolio of activities (or tasks) that all must be successfully completed before the risk is considered *closed*. Constructs, algorithms, and rules can be defined that provide program managers and decision-makers indices that measure the progress of the plan and isolate which activities or tasks, within the plan, are at risk for failure.

We begin with the following supposition: *A project's overall risk mitigation performance can be measured as a function of the collective performance of its risk management plans* (*RMPs*). The performance of an RMP can be measured as a function of the collective performance of its activities. The performance of the activities can be measured as a function of whether they are on track to meet their stated objectives or goals.

On-track can be thought of as a *schedule measure*. Whether they will meet their stated objectives or goals can be thought of as a *probability of success measure*.

These measures can be logically rolled up through a hierarchical structure. This produces performance indices across and at various levels in the hierarchy, which itself is a model of a project's overall risk mitigation strategy. This is illustrated in Figure 4.26.

A Value Function Approach

Next, value functions can be developed to evaluate the performance of each activity (in a risk management plan) with respect to its schedule status and its success probability. In this context, each activity is evaluated in terms of how well it is progressing along the planned schedule and the chance its objectives, goals, or outcomes will be successfully achieved.

Suppose the project team developed the following value function for an activity's schedule, measured as the *percent increase in planned duration* (PIPD).

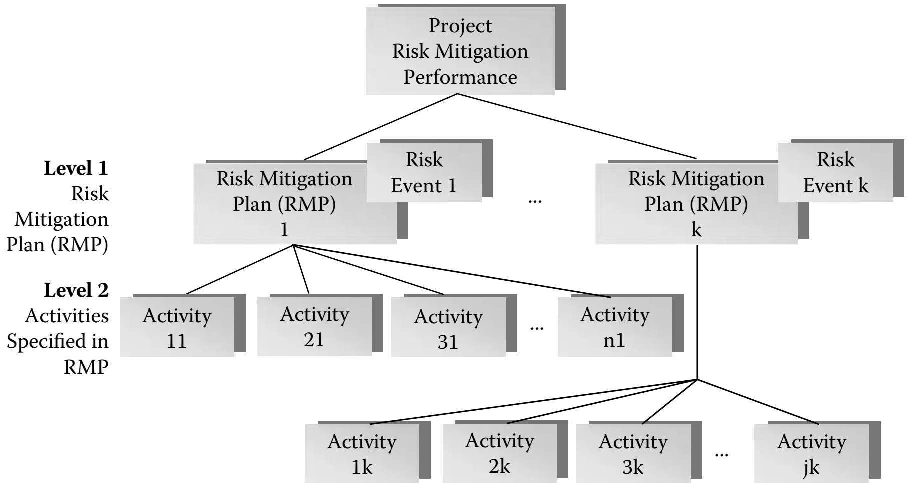


Figure 4.26: A portfolio view of risk mitigation plans and activities.

This function is given by Equation 4.57; specifically,

$$V_{PIPD}(x_1) = \begin{cases} 0 & \text{if } x_1 \leq 5 \\ 1.06(1 - e^{0.064(5-x_1)}) & \text{if } 5 < x_1 \leq 50 \end{cases} \tag{4.57}$$

Figure 4.27 is a plot of this equation. In addition, suppose the project team developed the following value function for an activity's success probability. This

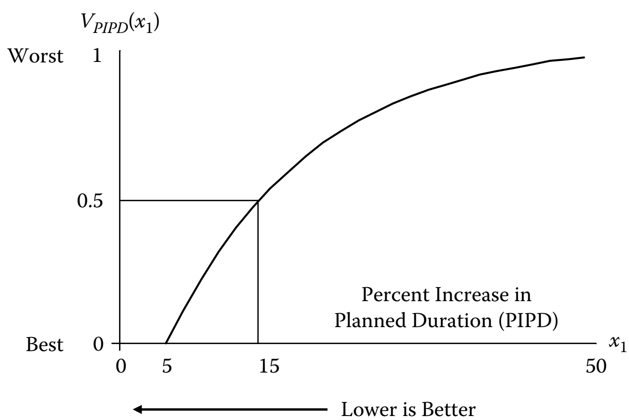


Figure 4.27: A value function for percent increase in planned duration.

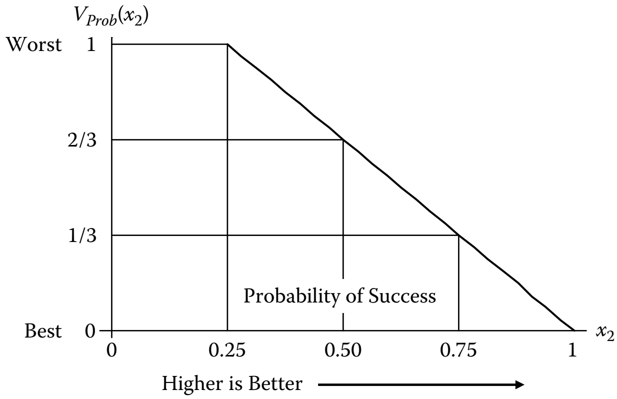


Figure 4.28: A value function for an activity's success probability.

function is given by Equation 4.58; specifically,

$$V_{Prob}(x_2) = \begin{cases} 1 & \text{if } x_2 \leq 0.25 \\ 4(1 - x_2)/3 & \text{if } 0.25 < x_2 \leq 1 \end{cases} \tag{4.58}$$

Figure 4.28 is a plot of this equation. Let's take a closer at these two value functions. The value function given by $V_{PIPD}(x_1)$ has the property that the lower the percent increase in planned duration the better. In particular, there is no "penalty" if the percent increase in planned duration is less than or equal to 5%. This is a threshold established by the project team and should be tailored for other projects. However, the penalty increases exponentially as an activity, within a risk management plan, slips in its planned duration by more than 5%. This increase is at a rate according to Equation 4.57.

The value function given by $V_{Prob}(x_2)$ has the property that the higher the probability of success the better. In particular, the penalty is greatest if an activity's success probability is less than or equal to 0.25. This is a threshold established by the project team and should be tailored for other projects. The penalty decreases linearly as an activity's success probability increases beyond 0.25.

Suppose the project team decided an activity is *closed only when it has successfully achieved its objectives, goals, or outcomes*. Let this be signaled by setting the activity's success probability equal to one. Consequently, an activity, within a risk management plan, is considered on-going, or *open*, if its success probability is not equal to one.

Suppose we define an activity's overall performance index by the following expression.

$$V_{API}(Activity) = \begin{cases} 1 & \text{if Probability of Success } \leq 0.25 \\ u_1 V_{PIPD}(x_1) + u_2 V_{Prob}(x_2) & \text{otherwise} \end{cases} \quad (4.59)$$

where u_1 and u_2 are non-negative weights that sum to one. Let the term V_{API} denote the *Activity Performance Index* (API) of a specific activity contained within a project's risk management plan. Finally, we define

$$V_{AAPI}(Activity) = \begin{cases} 0 & \text{if Probability of Success } = 1 \\ V_{API}(Activity) & \text{otherwise} \end{cases} \quad (4.60)$$

where V_{AAPI} is the *Adjusted Activity Performance Index* (AAPI). The AAPI is equal to zero only when the activity has achieved its objectives, goals, or outcomes (i.e., successfully completed) and is equal to $V_{API}(Activity)$ otherwise.

Suppose, for purposes of this discussion, an activity's success probability was considered by the project team to be twice as important as its schedule progress. Given this, Equation 4.59 takes the specific form given by Equation 4.61.

$$V_{API}(Activity) = \begin{cases} 1 & \text{if Probability of Success } \leq 0.25 \\ \frac{1}{3} V_{PIPD}(x_1) + \frac{2}{3} V_{\text{Prob}}(x_2) & \end{cases} \quad (4.61)$$

From this, an activity's progress can be measured at specified time steps, say t_0, t_1, t_2, ..., and their values tracked, accordingly. This enables project management to develop a time-history view of risk mitigation progress and a way to isolate those activities in a risk management plan, or across a set of plans, that are lagging in performance.

Case Discussion 4.2 Suppose a risk management plan is made up of three activities, as shown Figure 4.29. Suppose Table 4.15 summarizes the performance assessments of Activity 11.

In Table 4.15, the assessment dates are shown in the left-most column. The next column is the percent increase in planned duration (PIPD) at the assessment date shown. The third column is the assessment of Activity 11's success probability, at the assessment date shown. Given these latter two inputs, the next four columns

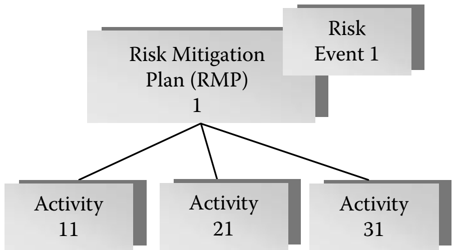


Figure 4.29: A risk management plan with three activities.

drive toward computing the Activity 11's overall performance index, at time t. For example, at assessment date t_0 we have the following from Equations 4.57 and 4.58, respectively.

$$V_{PIPD}(x_1 = 0 \mid t_0) = 0$$
$$V_{Prob}(x_2 = 0.95 \mid t_0) = 4(1 - 0.95)/3 = 0.067$$

From these values, we can determine Activity 11's performance index at assessment date t_0; that is, from Equation 4.61 we have

$$V_{API}(Activity\ 11 \mid t_0) = \frac{1}{3}V_{PIPD}(x_1 = 0 \mid t_0) + \frac{2}{3}V_{Prob}(x_2 = 0.95 \mid t_0) = 0.044$$

Furthermore, from Equation 4.60, we also have the following:

$$V_{AAPI}(Activity\ 11 \mid t_0) = V_{API}(Activity\ 11 \mid t_0)$$

Similar computations are made for the other assessment dates in Table 4.15. Next, suppose the other two activities in the risk management plan have the performance data shown in Tables 4.16 and 4.17, respectively.

TABLE 4.15: Activity 11 Assessments

Activity 11 Assessment Date	PIPD %	Prob Success	Value Function PIPD	Value Function Prob Success	API	AAPI
t0	0.00	0.95	0.000	0.067	0.044	0.044
t1	5.00	0.90	0.000	0.133	0.089	0.089
t2	7.50	0.90	0.156	0.133	0.141	0.141
t3	8.00	0.95	0.185	0.067	0.106	0.106
t4	9.00	0.95	0.239	0.067	0.124	0.124
t5	9.50	1.00	0.265	0.000	0.088	0.000

TABLE 4.16: Activity 21 Assessments

Activity 21 Assessment Date	PIPD	Prob Success	Value Function PIPD	Value Function Prob Success	API	AAPI
$t0$	0.00	0.75	0.000	0.333	0.222	0.222
$t1$	0.00	0.85	0.000	0.200	0.133	0.133
$t2$	0.00	0.95	0.000	0.067	0.044	0.044
$t3$	10.00	0.75	0.290	0.333	0.319	0.319
$t4$	11.00	0.85	0.337	0.200	0.246	0.246
$t5$	12.00	0.95	0.382	0.067	0.172	0.172
$t6$	13.00	0.95	0.424	0.067	0.186	0.186
$t7$	14.00	1.00	0.463	0.000	0.154	0.000

From these data, we can plot the time-history of the progress of each activity in this risk's risk management plan. These plots are shown in Figure 4.30.

Next, we might want to determine the performance of the risk management plan (refer to Figure 4.29) given the performance of its three activities. Here, we need to aggregate the individual activity performance measures into an overall performance measure of the risk management plan. There are many ways this measure could be computed. For illustrative purposes two approaches are discussed below.

TABLE 4.17: Activity 31 Assessments

Activity 31 Assessment Date	PIPD	Prob Success	Value Function PIPD	Value Function Prob Success	API	AAPI
t0	0.00	0.50	0.000	0.667	0.444	0.444
t1	0.00	0.50	0.000	0.667	0.444	0.444
t2	5.00	0.33	0.000	0.893	0.596	0.596
t3	7.00	0.33	0.127	0.893	0.638	0.638
t4	10.00	0.33	0.290	0.893	0.692	0.692
t5	15.00	0.33	0.500	0.893	0.762	0.762
t6	20.00	0.50	0.653	0.667	0.662	0.662
t7	22.00	0.50	0.702	0.667	0.678	0.678
t8	30.00	0.75	0.845	0.333	0.504	0.504
t9	35.00	0.95	0.904	0.067	0.346	0.346
t10	40.00	1.00	0.946	0.000	0.315	0.000

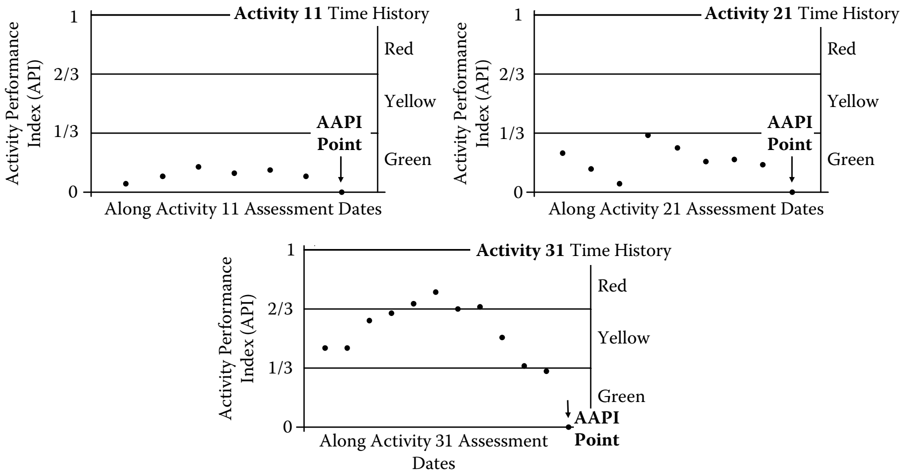


Figure 4.30: Activity Performance Index Time-Histories.

A Risk Management Plan Performance Index (RMPPI)

One way to define the performance index of a risk management plan is by a weighted average of its individual activity performance indices over time. So, given the data in Tables 4.15, 4.16, and 4.17 this rule would produce, at time t_0, an RMPPI equal to the following:

$$RMPPI \mid t_0 = \frac{1}{3}V_{API}(Activity\ 11 \mid t_0) + \frac{1}{3}V_{API}(Activity\ 21 \mid t_0) + \frac{1}{3}V_{API}(Activity\ 31 \mid t_0)$$

$$RMPPI \mid t_0 = \frac{1}{3}(0.044) + \frac{1}{3}(0.222) + \frac{1}{3}(0.444) = 0.237$$

So, at time t_0 we say the risk management plan's performance index is 0.237. This is not too bad a start. In the above, we've assumed each activity is equally important, hence the equal weighting. Unequal weights could be applied to emphasize the importance of one activity over another, if that consideration is appropriate or desired by the project's management. This same computation process is repeated for subsequent time periods to derive an overall time-history trend of the RMPPI. Management then monitors this trend to isolate where activity performance is proceeding well and where it is falling short of goals or outcomes.

A max average rule might be used instead of a weighted average rule to determine a risk management plan's performance index; that is, we could define RMPPI to be as follows:

$$\begin{aligned} RMPPI \mid t_0 &= \lambda Max\{V_{API}(Activity\,11 \mid t_0), V_{API}(Activity\,21 \mid t_0), \\ &\quad V_{API}(Activity\,31 \mid t_0)\} + (1-\lambda)Average\{V_{API}(Activity\,11 \mid t_0), \\ &\quad V_{API}(Activity\,21 \mid t_0), V_{API}(Activity\,31 \mid t_0)\} \end{aligned}$$

where λ is given (say) by Equation 4.14; that is,

$$0 < \lambda = 1 - \frac{1}{1+e^{10(m-1/2)}} < 1$$

where

$$m = Max\{V_{API}(Activity\,11 \mid t_0), V_{API}(Activity\,21 \mid t_0), V_{API}(Activity\,31 \mid t_0)\}.$$

Using this approach and given the data in Tables 4.15, 4.16, and 4.17, a max average rule would produce, at time t_0, and RMPPI equal to the following:

$$RMPPI \mid t_0 = \lambda Max\{0.044, 0.222, 0.444\} + (1-\lambda)Average\{0.044, 0.222, 0.444\}$$

where, in this case, $\lambda = 0.3635$. From this it follows that

$$\begin{aligned} RMPPI \mid t_0 &= \lambda Max\{0.044, 0.222, 0.444\} + (1-\lambda)Average\{0.044, 0.222, 0.444\} \\ &= 0.3635(0.444) + 0.6365(0.237) = 0.312 \end{aligned}$$

This same computation process is repeated for subsequent time periods to derive an overall time-history trend of the RMPPI. Management monitors this trend to isolate where performance is proceeding well and where it is falling short of goals or outcomes. Table 4.18 summarizes these computations. Figure 4.31 illustrates a time-history trend of the RMPPI, in this case discussion. The average and max average rules are shown.

This discussion demonstrated the development of an index for a single RMP as a function of its individual activity performance indices. What about the case of multiple RMPs? Can an overall project-level risk mitigation performance index be developed — one that management can track at the project's top-most level?

One way to address this is to consider a project-level risk mitigation performance index as a function of the performance indices of a project's RMPs as shown in Figure 4.32.

Here, various rules can be applied to derive an overall project risk mitigation performance index as a "rollup" of the performance indices of the project's RMPs.

TABLE 4.18: Illustrative RMPPI Computations

	Average		Max Average	
Assessment Date	**RMPPI API**	**RMPPI AAPI**	**RMPPI API**	**RMPPI AAPI**
$t0$	0.2370	0.2370	0.3127	0.3127
$t1$	0.2222	0.2222	0.3032	0.3032
$t2$	0.2603	0.2603	0.5024	0.5024
$t3$	0.3542	0.3542	0.5808	0.5808
$t4$	0.3539	0.3539	0.6489	0.6489
$t5$	0.3407	0.3113	0.7337	0.7317
$t6$	0.3120	0.2826	0.6043	0.5995
$t7$	0.3070	0.2261	0.6249	0.6133
$t8$	0.2488	0.1680	0.3788	0.3392
$t9$	0.1961	0.1152	0.2224	0.1558
$t10$	0.1860	0.0000	0.2037	0.0000

Possible rollup rules could again be the average of the RMPPIs across all RMPs at date t or the max average of the RMPPIs across all RMPs at date t.

4.4.3 Allocating Resources — A Simple Knapsack Model

This section illustrates one approach for allocating risk mitigation resources to risks considered most critical to address. This topic falls into the domain of *resource allocation problems*, a topic in the field of *operations research*.

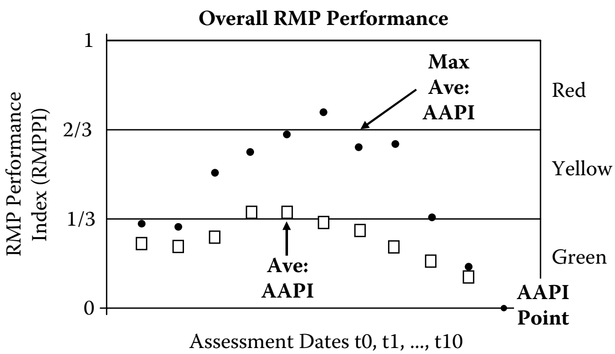


Figure 4.31: RMPPI time-history plot.

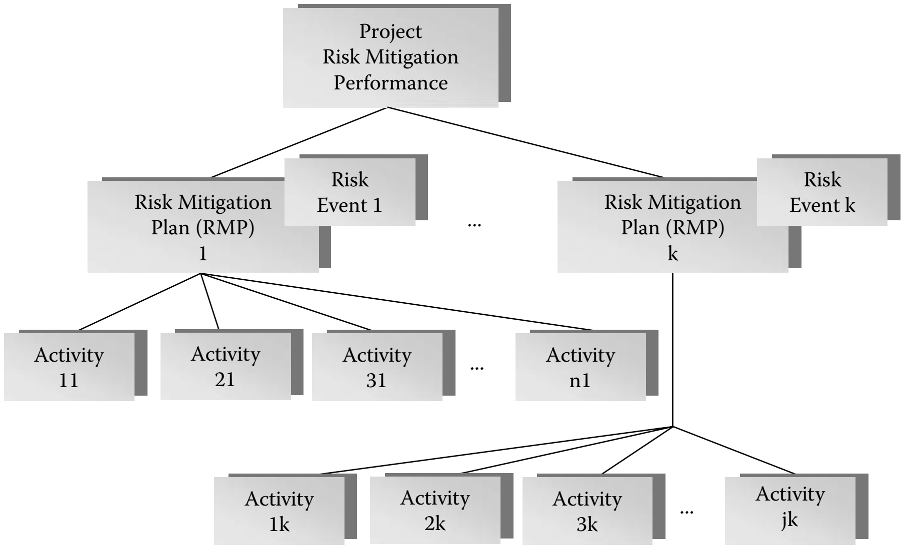


Figure 4.32: A project risk mitigation performance index.

The approach will demonstrate a mathematical technique developed to solve the *knapsack problem*. We will show how to set up the solution to this problem using the Solver feature of the Microsoft Excel® program. The knapsack problem is a classic problem in operations research. One form of this problem can be described as follows.

Suppose you have a finite collection of items you want to pack into your knapsack. Suppose the knapsack has limited capacity so it is not possible to include all items. Suppose each item has a certain value (or utility) to you. Given this, which items can be included in the knapsack such that the value of its collection of items is maximized but does not exceed the knapsack's capacity?

A Knapsack Problem Formulation

A classic knapsack problem can be mathematically formulated as follows:

$$\text{Maximize } v_1x_1 + v_2x_2 + v_3x_3 + \cdots + v_nx_n$$

$$\text{subject to } w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n \leq K$$

where x_i for $i = 1, 2, 3, \ldots, n$ takes the value 0 if item x_i is *not* included in the knapsack and takes the value 1 if item x_i *is* included in the knapsack. The parameter w_i is the weight (e.g., in pounds) of item x_i and K is the overall weight capacity of the knapsack. The first equation is called the objective function. The second equation is called the constraint (or constraint equation).

Solving this problem involves *integer programming* — a specialized operations research optimization technique. The theory of integer programming is beyond the scope of this book. However, we present a practical way of solving integer programming problems. For this, we will use the Microsoft Excel® Solver program and show how it applies to these problem contexts.

Suppose we want to solve the following knapsack problem.

$$\text{Maximize } 8x_1 + 12x_2 + 16x_3 + 24x_4$$
$$\text{subject to } 5x_1 + 8x_2 + 12x_3 + 17x_4 \leq 25$$

where x_i for $i = 1, 2, 3, 4$ takes the value 0 if item x_i is *not* included in the knapsack and takes the value 1 if item x_i *is* included in the knapsack. The coefficients 8, 12, 16, and 24 represent the value (or utility) of item x_1, x_2, x_3, and x_4, respectively. The coefficients 5, 8, 12, and 17 represent the weight (in pounds) of item x_1, x_2, x_3, and x_4, respectively. If the knapsack can hold up to 25 pounds, what is the optimal collection of items to include in the knapsack without exceeding its capacity?

Table 4.19 is a representation of the above problem in a format that can be entered into Excel. Row 2 represents the binary 0-1 decision variables; that is, the x_i's. Row 3 contains the coefficients of the objective function; that is, the function we wish to maximize. Row 4 contains the coefficients of the constraint equation. The last two right-most columns in Table 4.19 denote the left-hand side (LHS) and the right-hand side (RHS) of the objective function and the constraint equation. The LHS for the objective function is 60 when the x_i's all equal one. Similarly, the LHS for the constraint equation is 42 when the x_i's all equal one. These two values should be computed using the "sum-product" rule in Excel. Finally, notice

TABLE 4.19: An Excel Table of Input Values to Solver

	A	**B**	**C**	**D**	**E**		
1	**Input Matrix**	Item 1	Item 2	Item 3	Item 4	LHS	RHS
2	Decision Variables	1	1	1	1		
3	Objective Function	8	12	16	24	60	
4	Constraint	5	8	12	17	42	25

TABLE 4.20: Solver Solution to the Input Matrix in Table 4.19

	A	B	C	D	E		
1	**Input Matrix**	Item 1	Item 2	Item 3	Item 4	LHS	RHS
2	Decision Variables	1	1	1	1		
3	Objective Function	8	12	16	24	60	
4	Constraint	5	8	12	17	42	**25**
1	**Solution Matrix**	Item 1	Item 2	Item 3	Item 4	LHS	RHS
2	Decision Variables	1	1	1	0		
3	Objective Function	8	12	16	24	36	
4	Constraint	5	8	12	17	25	**25**

the value shown in Table 4.19 for RHS. This value reflects the capacity weight of the knapsack; that is, $K = 25$ pounds in this case.

Table 4.20 presents the solution matrix that results from running the Solver program on the data in Table 4.19. Here, the mix of items to select for the knapsack that optimizes the objective function, while not exceeding the weight constraint of 25 pounds, is item 1, item 2, and item 3. Notice the objective function is maximized at a value of 36. This is the largest value this function can take given it must also satisfy the constraint of not exceeding the knapsack's weight limit of 25 pounds.

Application to Risk Mitigation Resource Allocation

Methods used to select which items to place in a knapsack can also be used to choose which risks to fund as a function of their criticality and a fixed risk mitigation resource or budget. Here, we illustrate a solution to this decision problem and show its similarity to the above discussion.

Instead of a knapsack, let's think of the problem of choosing which risks to include in a *portfolio*. Here, the portfolio is defined by a fixed budget for funding risk mitigation plans and activities. The decision problem is to select those risks that have potentially the highest impacts to the project while also not exceeding the risk mitigation budget.

We can think of this as a knapsack problem. The mathematical set up is as follows. Let $j = \{1, 2, 3, \ldots, n\}$ be a set indexing the candidate risks. Let

$$x_j = \begin{cases} 0 & \text{if } j\text{th risk } \textit{is not} \text{ in the portfolio} \\ 1 & \text{if } j\text{th risk } \textit{is in} \text{ the portfolio} \end{cases}$$

Here, we want to

$$\text{Maximize } v_1x_1 + v_2x_2 + v_3x_3 + \cdots + v_nx_n$$
$$\text{subject to } c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n \leq C$$

where v_j is the impact score of jth risk event (refer to section 4.3.2), c_j is the cost to mitigate the jth risk event, and C is the total risk mitigation cost budget.

Table 4.21 illustrates this application context for 10 risk events. Suppose the top half of the table is a project's "top-10" risks. Suppose all are competing for limited risk mitigation resources. Suppose the total risk mitigation budget for this project is \$1 million; however, the cost to mitigate all 10 risk events is just under \$2 million. Given this, which risks should be included in the "risk mitigation portfolio" such that we maximize mitigating those risks with the highest potential project impacts while not exceeding the \$1 million budget?

TABLE 4.21: A Risk Input Matrix and Solver Solution

Input Matrix	**Risk Event 1**	**Risk Event 2**	**Risk Event 3**	**Risk Event 4**	**Risk Event 5**		
Decision Variables	1	1	1	1	1		
Objective Function	0.70	0.84	0.79	0.93	0.67		
Constraint	290	132	234	178	100		
	Risk Event 6	**Risk Event 7**	**Risk Event 8**	**Risk Event 9**	**Risk Event 10**	**LHS**	**RHS**
Decision Variables	1	1	1	1	1		
Objective Function	0.88	0.95	0.90	1.00	0.82	8.48	
Constraint	145	189	223	123	357	1971	1000
Solution Matrix	**Risk Event 1**	**Risk Event 2**	**Risk Event 3**	**Risk Event 4**	**Risk Event 5**		
Decision Variables	0	1	0	1	0		
Objective Function	0.70	0.84	0.79	0.93	0.67		
Constraint	290	132	234	178	100		
	Risk Event 6	**Risk Event 7**	**Risk Event 8**	**Risk Event 9**	**Risk Event 10**	**LHS**	**RHS**
Decision Variables	1	1	1	1	0		
Objective Function	0.88	0.95	0.90	1.00	0.82	5.50	
Constraint	145	189	223	123	357	990	1000

To find the optimal collection of risks to include in the portfolio we can model this situation as a "knapsack" problem. In Table 4.21, the coefficients of the objective function are the impact scores of the risk events. The coefficients of the constraint equation are the mitigation costs (in dollars-thousands) of the risk events. For example, risk event 1 has a project impact of 0.70 and a mitigation cost of $290,000 dollars. Using the Solver program, the risk events to include in the mitigation portfolio are indicated in the lower half of Table 4.21. These are the events indicated by a "1" in the solution matrix. Here, risk events 2, 4, 6, 7, 8, and 9 are the *optimal* mix of risks to include in the portfolio. This mix is the "best" combination of risks to include in the portfolio that (1) offers the largest reduction in potential project impacts and (2) does not exceed the total risk mitigation budget of $1 million.

In summary, the approach described in this section illustrates a formal way to allocate limited risk mitigation resources to events considered most critical to address on a project. Analytical approaches, such as these, are valuable "first-filters" that *support* decision-making. They are not replacements for human judgment or creative project management. Leadership should always look at results, such as those generated in Table 4.21, and consider other tradeoffs, options, or creative ways to address critically impacting risks given a limited risk mitigation budget. One way to use this analysis is to let it form the basis for arguing that increased resources are needed. It reveals not only those risks that can be included in the portfolio but also those that cannot be included, if the budget constraint is maintained.

Next, we switch to another important topic in systems engineering risk management. This is the use of technical performance measures as indicators of a system's overall performance risks. Shown will be ways to use these measures to monitor the progress of technical performance and risks to performance faced by an engineering system.

4.5 Measuring Technical Performance Risk

... You have to reach a level of comfort with that risk.

Sally Ride (U.S. Astronaut)

Technical Performance Measures (TPMs) are traditionally defined and evaluated to assess how well a system is achieving its performance requirements. Typically, dozens of TPMs are defined for a system. Although they generate useful information and data about a system's performance, little is available in the program and engineering management communities on how to integrate these measures into a meaningful measure of a system's overall performance risk.

This section presents how individual TPMs can be combined to measure and monitor the overall performance risk of a system. The approach consists of integrating individual technical performance measures in a way that produces an overall risk index. The computed index shows the degree of performance risk presently in the system. It identifies risk-driving TPMs, enables monitoring time-history trends, and reveals where management should target strategies to lessen or eliminate the performance risks of the system.

4.5.1 A Technical Performance Risk Index Measure

As a system evolves through its engineering phases, management defines and derives measures that indicate how well the system is achieving its performance requirements. These measures are known as *Technical Performance Measures* (TPMs) [6, 7, 8]. Measures such as *Weight, Mean-Time-Between-Failure*, and *Detection Accuracy* represent the types of TPMs often defined on programs.

Technical performance measurements can be taken from a variety of sources. This includes data from system testing, system simulations, or experimentations. Depending on the source basis for these data, and the development phase of the system, performance data may be derived from a mix of actual or forecast values.

Mentioned above, the program and engineering management communities have little in the way of methodology for quantifying performance risk as a function of a system's individual technical performance measures. The approach presented herein consists of computing a risk index derived from these individual performance measurements. The index shows the degree of performance risk presently in the system, supports identifying risk-driving TPMs, and can reveal where management should focus on improving technical performance and, thereby, lessen risk. When the index is continuously updated, management can monitor the time-history trend of its value. This enables management to assess the effectiveness of risk reduction actions being targeted or achieved over time.

In general, TPMs are measures that, when evaluated over time, must either decrease to meet a system's performance requirements or increase to meet performance requirements. Thus, each TPM can be assigned to one of two categories.

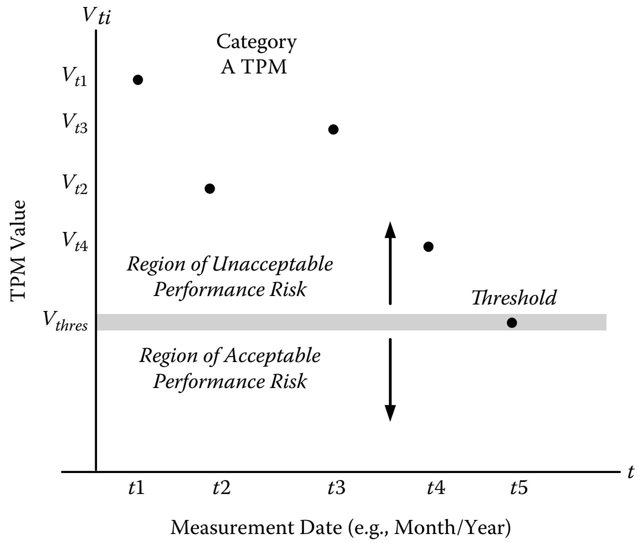


Figure 4.33: A Category A TPM.

Define *Category A* as the collection of TPMs whose values must decrease to achieve a system's threshold performance requirements. Define *Category B* as the collection of TPMs whose values must increase to achieve a system's threshold performance requirements. This is illustrated in Figure 4.33 and Figure 4.34.

In Figures 4.33 and 4.34 the horizontal axes represents a measurement date. This is the date when the actual or forecasted value of the TPM was taken or derived.

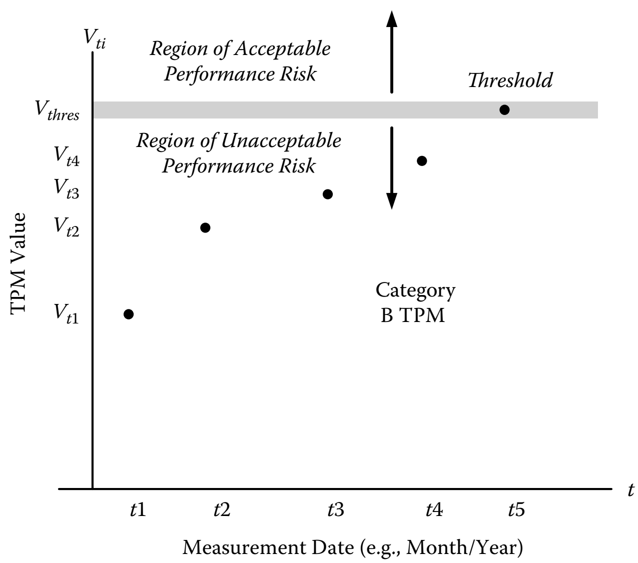


Figure 4.34: A Category B TPM.

In Figures 4.33 and 4.34 the vertical axes represents the value of the TPM at the corresponding measurement date.

In Figures 4.33 and 4.34 V_{thres} denotes the threshold performance value for the TPM. This is the minimum acceptable value for the TPM. It marks the boundary between the regions of acceptable versus unacceptable performance risk.

It is assumed that TPMs defined for a system are done judiciously; that is, only those TPMs truly needed to properly measure a system's overall technical performance are defined, measured, and monitored. Given this, *acceptable performance risk* can be defined as the condition when all TPMs reach, or extend beyond, their individual threshold performance values. Conversely, *unacceptable performance risk* can be defined as the condition when one or more TPMs have not reached their individual threshold performance values.

A Technical Performance Risk Index Measure

The following presents an index designed to measure the performance risk of a system. The index provides a numerical indicator of how well a developing system is progressing toward its threshold performance requirements. It serves as a yardstick that enables management to measure the "distance" the system is from its minimum performance thresholds and to monitor trends over time.

To develop the risk index, it is necessary to normalize the TPM "raw" values into a common and dimensionless scale. Figures 4.35 through 4.38 show such scales for Category A and Category B TPMs. In these figures, the top-most vertical scales reflect TPM raw values (their native units) taken from measurements, tests, experiments, or prototypes. The bottom-most vertical scales reflect TPM normalized values. Here, threshold values are all normalized to one. This scale transformation is done for each TPM in each category. This allows management to compare the progress of each performance measure in a common and dimensionless scale. From these normalized scales, an overall measure of the extent to which the performance of the system meets its threshold requirements can be determined. Next are formulas to derive this measure. This is followed by a computation example to illustrate the application context.

Mentioned previously, let Category A be the set of TPMs that need to be reduced to their threshold values. In Figure 4.35, let $V_{ti,\, \mathrm{A}j}$ be the value at time ti for the jth TPM in Category A and $V_{thres,\, \mathrm{A}j}$ be the threshold value to which the jth TPM is driven. In Figure 4.36, define $v_{ti,\, \mathrm{A}j}$ to be a normalized TPM value against

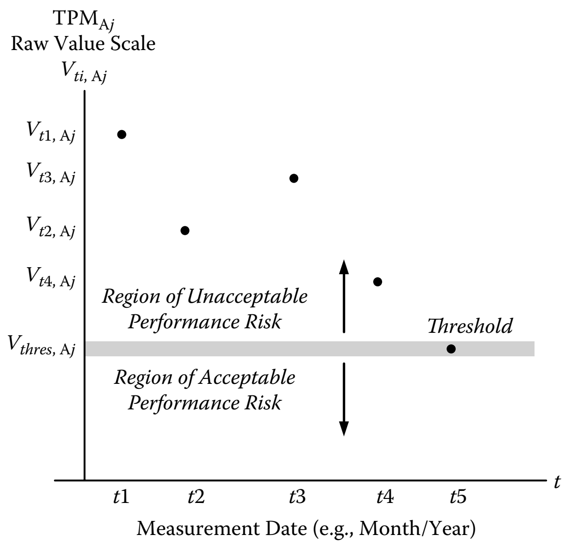


Figure 4.35: Category A TPM raw values.

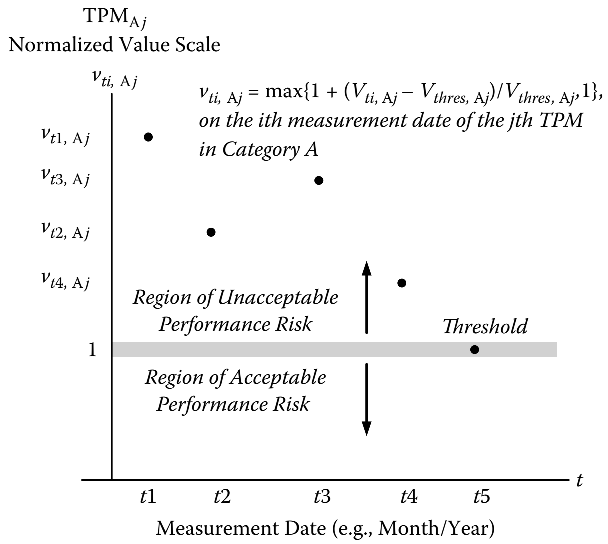


Figure 4.36: Category A TPM normalized values.

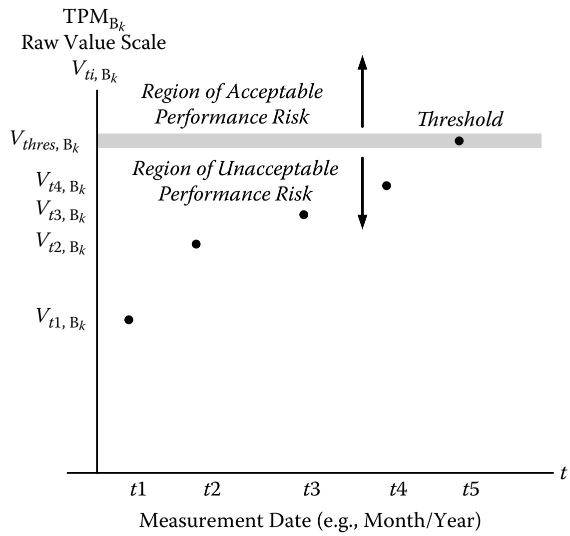


Figure 4.37: Category B TPM raw values.

its threshold as follows (assuming both $V_{ti,\mathrm{A}j}$ and $V_{thres,\mathrm{A}j}$ are greater than zero):

$$\begin{aligned} v_{ti,\mathrm{A}j} &= \max\{V_{ti,\mathrm{A}j}, V_{thres,\mathrm{A}j}\}/V_{thres,\mathrm{A}j} \text{ (i.e., threshold met if } V_{ti,\mathrm{A}j} \leq V_{thres,\mathrm{A}j}) \\ &= \max\{V_{ti,\mathrm{A}j}/V_{thres,\mathrm{A}j}, 1\} \\ &= \max\{(V_{thres,\mathrm{A}j} - V_{thres,\mathrm{A}j} + V_{ti,\mathrm{A}j})/V_{thres,\mathrm{A}j}, 1\} \\ &= \max\{1 + (V_{ti,\mathrm{A}j} - V_{thres,\mathrm{A}j})/V_{thres,\mathrm{A}j}, 1\} \ (\geq 1) \end{aligned} \tag{4.62}$$

Equation 4.62 is the formula for $v_{ti,\mathrm{A}j}$ in Figure 4.36, which brings out the overage above 1. Similarly, let Category B be the set of TPMs that need to be increased to their threshold values. In Figure 4.37, let $V_{ti,\mathrm{B}k}$ be the value at time *ti* for the *k*th TPM in Category B and $V_{thres,\mathrm{B}k}$ be the threshold value to which the *k*th TPM is driven. In Figure 4.38, define $v_{ti,\mathrm{B}k}$ to be a normalized TPM value against its threshold as follows (assuming both $V_{ti,\mathrm{B}k}$ and $V_{thres,\mathrm{B}k}$ are greater than zero):

$$\begin{aligned} v_{ti,\mathrm{B}k} &= \min\{V_{ti,\mathrm{B}k}, V_{thres,\mathrm{B}k}\}/V_{thres,\mathrm{B}k} \text{ (i.e., threshold met if } V_{ti,\mathrm{B}k} \geq V_{thres,\mathrm{B}k}) \\ &= \min\{V_{ti,\mathrm{B}k}/V_{thres,\mathrm{B}k}, 1\} \\ &= \min\{(V_{thres,\mathrm{B}k} - V_{thres,\mathrm{B}k} + V_{ti,\mathrm{B}k})/V_{thres,\mathrm{B}k}, 1\} \\ &= \min\{1 - (V_{thres,\mathrm{B}k} - V_{ti,\mathrm{B}k})/V_{thres,\mathrm{B}k}, 1\} \ (\leq 1) \end{aligned} \tag{4.63}$$

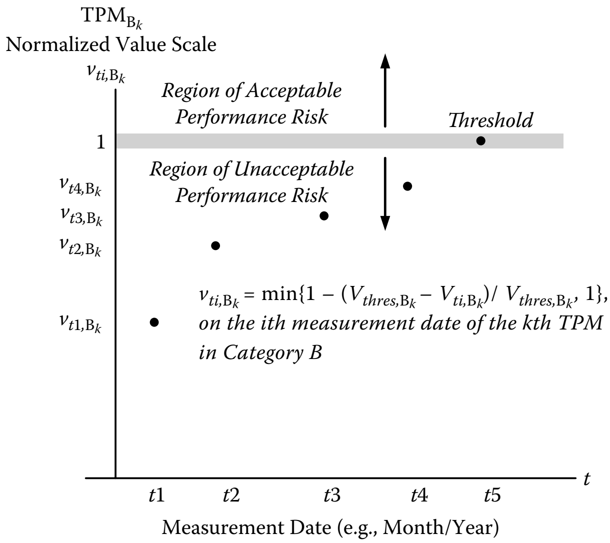


Figure 4.38: Category B TPM normalized values.

Equation 4.63 is the formula for $v_{ti,Bk}$ in Figure 4.38, which brings out the underage below one. From the normalized values, we now calculate their average difference from one for each category and use it as the category's *TPM Risk Index* (*TRI*). Assume $j = 1, 2, \ldots, m$ for Category A (m-elements) and $k = 1, 2, \ldots, n$ for Category B (n-elements). It follows that:

$$\begin{aligned} \text{TRI}_{ti,\text{A}} &= [(v_{ti,\text{A}1} - 1) + (v_{ti,\text{A}2} - 1) + \cdots + (v_{ti,\text{A}m} - 1)]/m \\ &= [(v_{ti,\text{A}1} + v_{ti,\text{A}2} + \cdots + v_{ti,\text{A}m})/m] - 1 \end{aligned} \tag{4.64}$$

$$\begin{aligned} \text{TRI}_{ti,\text{B}} &= [(1 - v_{ti,\text{B}1}) + (1 - v_{ti,\text{B}2}) + \cdots + (1 - v_{ti,\text{B}n})]/n \\ &= 1 - [(v_{ti,\text{B}1} + v_{ti,\text{B}2} + \cdots + v_{ti,\text{B}n})/n] \end{aligned} \tag{4.65}$$

These two indices show the average overage or underage for TPMs in Category A or Category B when their individual threshold values are re-scaled to 1. To combine all normalized values into an overall risk index, we first convert the TPMs in Category A into equivalent ones in Category B. This is because the normalized values for Category A can differ in orders of magnitude from those for Category B (e.g., 1000 vs. 0.5). An overall index, based on the normalized values as calculated, will be unduly influenced by large values. The result, though correct, can be difficult to interpret.

To make such a conversion, observe that for the jth TPM in Category A with value $V_{ti,\mathrm{A}j}$ and threshold $V_{thres,\mathrm{A}j}$, an equivalent TPM in Category B can be constructed with value $U_{ti,\mathrm{A}j} = 1/V_{ti,\mathrm{A}j}$ and threshold $U_{thres,\,\mathrm{A}j} = 1/V_{thres,\,\mathrm{A}j}$. Typically, the reciprocal of a TPM is just as practical. For example, a failure rate or a processing delay that is to be reduced can be taken in its reciprocal respectively as a mean time between failure or a completion rate that is to be increased.

The probability of a certain undesirable event (e.g., misclassification or an error exceeding the tolerance) or unavailability of a certain desirable state (e.g., system working or parts-in-hand) are more subtle. But their reciprocals can be viewed as the expected number of events that will contain one such undesirable event or the expected length of time that will contain one unit time of such a desirable state being unavailable. Although their complements (as opposed to reciprocals) can also be used as Category B TPMs, it is not recommended as the complements are usually close to one and their further improvements toward one do not show much difference when normalized.

The normalized value for a Category A TPM converted into a Category B TPM is, by definition

$$\begin{aligned} \mathrm{u}_{ti,\mathrm{A}j} &= \min\{U_{ti,Aj}, U_{thres,\,\mathrm{A}j}\}/U_{thres,\,\mathrm{A}j} \\ &= \min\{1/V_{ti,\mathrm{A}j}, 1/V_{thres,\,\mathrm{A}j}\}/(1/V_{thres,\,\mathrm{A}j}) \\ &= [1/\max\{V_{ti,\mathrm{A}j}, V_{thres,\,\mathrm{A}j}\}]/(1/V_{thres,\,\mathrm{A}j}) \\ &= 1/[\max\{V_{ti,\mathrm{A}j}, V_{thres,\,\mathrm{A}j}\}/V_{thres,\,\mathrm{A}j}] \\ &= 1/v_{ti,\mathrm{A}j}\,(\leq 1) \end{aligned} \tag{4.66}$$

We can now treat all TPMs as Category B and derive an overall risk index. Let

$$\mathrm{TRI}^*_{ti,\mathrm{A}} = 1 - [(u_{ti,\mathrm{A}1} + u_{ti,\mathrm{A}2} + \cdots + u_{ti,\,\mathrm{A}m})/m] \tag{4.67}$$

$$\mathrm{TRI}_{ti,\,\mathrm{B}} = 1 - [(v_{ti,\,\mathrm{B}1} + v_{ti,\,\mathrm{B}2} + \cdots + v_{ti,\,\mathrm{B}n})/n] \tag{4.68}$$

then

$$\begin{aligned} \mathrm{TRI}_{ti,\,\mathrm{All}} &= 1 - [(u_{ti,\mathrm{A}1} + u_{ti,\mathrm{A}2} + \cdots + u_{ti,\mathrm{A}m} + v_{ti,\,\mathrm{B}1} \\ &\qquad + v_{ti,\,\mathrm{B}2} + \cdots + v_{ti,\,\mathrm{B}n})/(m+n)] \\ &= 1 - [(m(1 - \mathrm{TRI}^*_{ti,\mathrm{A}}) + n(1 - \mathrm{TRI}_{ti,\,\mathrm{B}}))/(m+n)] \\ &= [m(\mathrm{TRI}^*_{ti,\mathrm{A}}) + n(\mathrm{TRI}_{ti,\,\mathrm{B}})]/(m+n) \end{aligned} \tag{4.69}$$

where $TRI_{ti,\,All}$ is the overall *TPM Risk Index* for the system, computed across all the system's TPMs. Finally, a non-negative weight w_{Aj} could be assigned to $(1 - u_{ti,Aj})$ for the jth TPM in Category A and w_{Bk} to $(1 - v_{ti,\,Bk})$ for the kth TPM in Category B (as opposed to all having an equal weight, as assumed in the discussion above). In that case, we have the following:

$$TRI^{*}_{ti,A} = 1 - [(w_{A1}u_{ti,A1} + w_{A2}u_{ti,\,A2} + \cdots + w_{Am}u_{ti,\,Am})/W_A] \qquad (4.70)$$

where

$$W_A = w_{A1} + w_{A2} + \cdots + w_{Am}$$

$$TRI_{ti,\,B} = 1 - [(w_{B1}v_{ti,\,B1} + w_{B2}v_{ti,\,B2} + \cdots + w_{Bn}v_{ti,\,Bn})/W_B] \qquad (4.71)$$

where

$$W_B = w_{B1} + w_{B2} + \cdots + w_{Bn}$$

and

$$TRI_{ti,\,All} = [W_A TRI^{*}_{ti,\,A} + W_B TRI_{ti,\,B}]/W \qquad (4.72)$$

In Equation 4.72, $W = W_A + W_B$. Thus, Equation 4.72 is the most general form of the system's overall *TPM Risk Index*.

From above, $TRI^{*}_{ti,\,A}$, $TRI_{ti,\,B}$, and $TRI_{ti,\,All}$, equally or unequally weighted, are bounded by zero and one. A value of zero for the risk indices means *there are no unacceptable risks* in the included TPMs, each achieving (or extending beyond) its threshold value. The risk indices can be asymptotically near one and that implies that each TPM value in Category A is very large when compared with its threshold or that each TPM value in Category B is very small when compared to its threshold (i.e., all far away from their thresholds). When the TPMs are moving toward their thresholds, the risk indices are moving toward zero.

Computation Example & Time-History Graph

Suppose Table 4.22 represents a system's set of Category A and Category B TPMs, along with their threshold and raw values for six measurement dates. From these data, what is the system's overall technical performance risk index? How is it changing over time?

From the data in Table 4.22 and Equations 4.70, 4.71, and 4.72, we can derive, for each measurement date, the TPM risk indices for the Category A and Category B TPMs, as well as for the system's overall *TPM Risk Index*. The results from these derivations are summarized in Table 4.23.

TABLE 4.22: Computational Example: Category A TPMs

Category A TPM	Vthres,A	Raw Value V(ti,A)	Eq. 4-62 v(ti, A)	Eq. 4-66 u(ti, A)	wt
Measurement Date t1					
Average Processing Delay (msecs)	1.000	3.000	3.000	0.333	1.000
Mean Time to Repair (mins)	10.000	50.000	5.000	0.200	1.000
Payload Weight (lbs)	950.000	2112.000	2.223	0.450	1.000
Time for Engagement Coordination (sec)	0.010	0.100	10.000	0.100	1.000
TRI*(t1,A)	**0.729**	**Eq. 4-70**			
Measurement Date t2					
Average Processing Delay (msecs)	1.000	2.860	2.860	0.350	1.000
Mean Time to Repair (mins)	10.000	43.000	4.300	0.233	1.000
Payload Weight (lbs)	950.000	1764.000	1.857	0.539	1.000
Time for Engagement Coordination (sec)	0.010	0.040	4.000	0.250	1.000
TRI*(t2,A)	**0.657**	**Eq. 4-70**			
Measurement Date t3					
Average Processing Delay (msecs)	1.000	1.180	1.180	0.847	1.000
Mean Time to Repair (mins)	10.000	43.000	4.300	0.233	1.000
Payload Weight (lbs)	950.000	1328.000	1.398	0.715	1.000
Time for Engagement Coordination (sec)	0.010	0.032	3.200	0.313	1.000
TRI*(t3,A)	**0.473**	**Eq. 4-70**			
Measurement Date t4					
Average Processing Delay (msecs)	1.000	1.090	1.090	0.917	1.000
Mean Time to Repair (mins)	10.000	27.000	2.700	0.370	1.000
Payload Weight (lbs)	950.000	1189.000	1.252	0.799	1.000
Time for Engagement Coordination (sec)	0.010	0.020	2.000	0.500	1.000
TRI*(t4,A)	**0.353**	**Eq. 4-70**			
Measurement Date t5					
Average Processing Delay (msecs)	1.000	1.030	1.030	0.971	1.000
Mean Time to Repair (mins)	10.000	12.000	1.200	0.833	1.000
Payload Weight (lbs)	950.000	1008.000	1.061	0.942	1.000
Time for Engagement Coordination (sec)	0.010	0.010	1.000	1.000	1.000
TRI*(t5,A)	**0.063**	**Eq. 4-70**			
Measurement Date t6					
Average Processing Delay (msecs)	1.000	0.980	1.000	1.000	1.000
Mean Time to Repair (mins)	10.000	9.000	1.000	1.000	1.000
Payload Weight (lbs)	950.000	948.000	1.000	1.000	1.000
Time for Engagement Coordination (sec)	0.010	0.010	1.000	1.000	1.000
TRI*(t6,A)	**0**	**Eq. 4-70**			

TABLE 4.22: Computational Example: Category A TPMs (*Continued*)

Category B TPM	Vthres,B	Raw Value V(ti,B)	Eq. 4-63 v(ti, B)	wt		
Measurement Date t1						
Interceptors Available (no. of units)	150.000	67.000	0.447	1.000		
Mean Time Between Failure (hours)	500.000	100.000	0.200	5.000		
Single Shot Success Probability (%)	0.950	0.870	0.916	1.000		
Damage Assessment Accuracy (%)	0.995	0.600	0.603	1.000		
Software Coding (no. of modules coded)	763.000	578.000	0.758	1.000		
TRI(t1,B)	**0.586**	**Eq. 4-71**			TRI(ti,All)	0.63
Measurement Date t2						
Interceptors Available (no. of units)	150.000	128.000	0.853	1.000		
Mean Time Between Failure (hours)	500.000	189.000	0.378	5.000		
Single Shot Success Probability (%)	0.950	0.890	0.937	1.000		
Damage Assessment Accuracy (%)	0.995	0.878	0.882	1.000		
Software Coding (no. of modules coded)	763.000	643.000	0.843	1.000		
TRI(t2,B)	**0.399**	**Eq. 4-71**			TRI(ti,All)	0.478
Measurement Date t3						
Interceptors Available (no. of units)	150.000	134.000	0.893	1.000		
Mean Time Between Failure (hours)	500.000	223.000	0.446	5.000		
Single Shot Success Probability (%)	0.950	0.910	0.958	1.000		
Damage Assessment Accuracy (%)	0.995	0.940	0.945	1.000		
Software Coding (no. of modules coded)	763.000	687.000	0.900	1.000		
TRI(t3,B)	**0.342**	**Eq. 4-71**			TRI(ti,All)	0.382
Measurement Date t4						
Interceptors Available (no. of units)	150.000	139.000	0.927	1.000		
Mean Time Between Failure (hours)	500.000	348.000	0.696	5.000		
Single Shot Success Probability (%)	0.950	0.934	0.983	1.000		
Damage Assessment Accuracy (%)	0.995	0.945	0.950	1.000		
Software Coding (no. of modules coded)	763.000	698.000	0.915	1.000		
TRI(t4,B)	**0.194**	**Eq. 4-71**			TRI(ti,All)	0.243
Measurement Date t5						
Interceptors Available (no. of units)	150.000	142.000	0.947	1.000		
Mean Time Between Failure (hours)	500.000	379.000	0.758	5.000		
Single Shot Success Probability (%)	0.950	0.940	0.989	1.000		
Damage Assessment Accuracy (%)	0.995	0.999	1.000	1.000		
Software Coding (no. of modules coded)	763.000	723.000	0.948	1.000		
TRI(t5,B)	**0.147**	**Eq. 4-71**			TRI(ti,All)	0.121
Measurement Date t6						
Interceptors Available (no. of units)	150.000	159.000	1.000	1.000		
Mean Time Between Failure (hours)	500.000	521.000	1.000	5.000		
Single Shot Success Probability (%)	0.950	0.990	1.000	1.000		
Damage Assessment Accuracy (%)	0.995	1.000	1.000	1.000		
Software Coding (no. of modules coded)	763.000	763.000	1.000	1.000		
TRI(t6,B)	**0**	**Eq. 4-71**			TRI(ti,All)	0

Note that TRI is a cardinal measure. This means its value is a measure of the "strength" or "distance" that the contributing TPMs are from their individual threshold performance values. A TRI equal to 0.5 is truly twice as "bad" as one equal to 0.25.

Figure 4.39 presents a time-history trend of the TPM risk indices for the data in Table 4.23. Here, the trend is good. All three TRIs are heading toward zero.

TABLE 4.23: TPM Risk Index Summaries

Measurement Date	TPM Risk Index for Category A TPMs $TRI^*_{ti,\,A}$ Eq. 4.70	TPM Risk Index for Category B TPMs $TRI_{ti,\,B}$ Eq. 4.71	Overall TPM Risk Index for the System $TRI_{ti,\,All}$ Eq. 4.72
$t1$	0.729	0.586	0.630
$t2$	0.657	0.399	0.478
$t3$	0.473	0.342	0.382
$t4$	0.353	0.194	0.243
$t5$	0.063	0.147	0.121
$t6$	0	0	0

This means all TPMs defined for the system are converging toward their individual threshold performance values. In practice, management should regularly produce a graphic summary such as this to monitor the extent each risk index is changing over time.

Summary

This discussion presented an approach and formalism for developing an overall set of quantitative indices that measure a system's performance risk, as a function of its TPMs. Below are the summary general forms of the three principal risk indices.

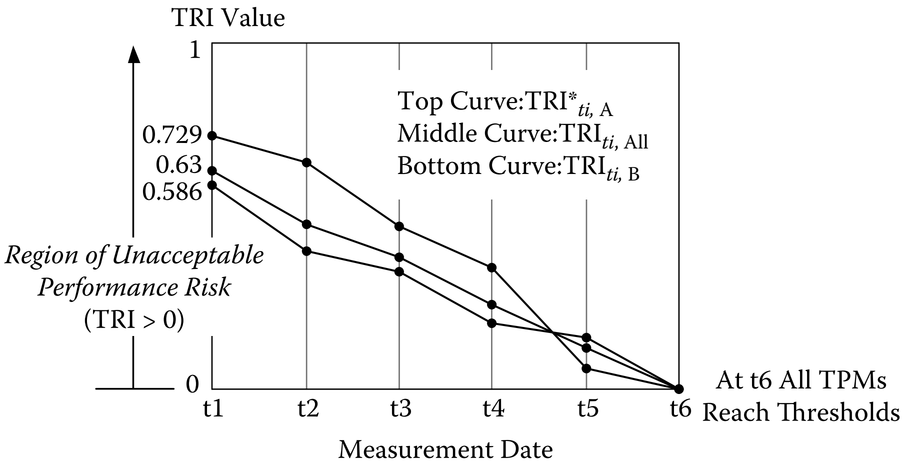


Figure 4.39: Illustrative TPM risk index time-history trend.

Category A. $TRI^*_{ti,\,A} = 1 - [(w_{A1}u_{ti,\,A1} + w_{A2}u_{ti,\,A2} + \cdots + w_{Am}u_{ti,\,Am})/W_A]$
where

$$W_A = w_{A1} + w_{A2} + \cdots + w_{Am}$$

Category B. $TRI_{ti,\,B} = 1 - [(w_{B1}v_{ti,\,B1} + w_{B2}v_{ti,\,B2} + \cdots + w_{Bn}v_{ti,\,Bn})/W_B]$
where

$$W_B = w_{B1} + w_{B2} + \cdots + w_{Bn}$$

Overall Risk Index. $TRI_{ti,\,All} = [W_A TRI^*_{ti,\,A} + W_B TRI_{ti,\,B}]/W$
where

$$W = W_A + W_B$$

To conclude, key features of the approach presented in this section are summarized as follows:

Provides Integrated Measures of Technical Performance: This approach provides management a way to transform dozens or more TPMs into common measurement scales. From this, all TPMs may be integrated and combined in a way that provides management with meaningful and comparative measures of the overall performance risk of the system, at any measurement time t.

Measures Technical Performance as a Function of the Physical Parameters of the TPMs: This approach operates on actual or predicted values from engineering measurements, tests, experiments, or prototypes. As such, the physical parameters that characterize the TPMs provide the basis for deriving the TPM risk indices.

Measures the Degree of Risk and Monitors Change over Time: The computed TPM risk indices show the degree of performance risk that presently exists in the system, supports the identification and ranking of risk-driving TPMs, and can reveal where management should focus on improving technical performance and, thereby, lessen risk. If the indices are continuously updated, then management can monitor the time-history trend of their values to assess the effectiveness of risk reduction actions being targeted or achieved over time.

Finally, note the TRI calculation assumes TPM threshold values as the goals that technical performance is driven to reach. The resulting index value measures the distance between the achieved technical performance levels and those considered minimally acceptable. One can use TPM objective values, the desirable but more demanding technical performance levels, to replace the threshold values in the TRI calculation. The result will be an index to measure the distance between the achieved levels and those considered desirable.

4.5.2 An Approach for Systems-of-Systems

This section extends the general formulation of the TRI to a system that is composed of many individual systems that, when connected, provide an overall system-of-systems capability. Here, we use the following definition of a system-of-systems.

A Definition*

"A system-of-systems (SoS) is a set or arrangement of interdependent systems that are related or connected to provide a given capability. The loss of any part of the system will degrade the performance or capabilities of the whole."

An example of an SoS could be interdependent information systems. While individual systems within the SoS may be developed to satisfy the peculiar needs of a given user group the information they share is so important that the loss of a single system may deprive other systems of the data needed to achieve even minimal capabilities.

A System-of-Systems Hierarchy

Shown in Figure 4.40, a system-of-systems can be decomposed into its individual systems. Next, these individual systems can be decomposed into their individual subsystems. The result of this process produces a tree-like hierarchical structure.

Here, each element in the hierarchy is referred to as a "node." The top-most node is called the *root node*. In Figure 4.40, the root node represents the SoS level. A *parent node* is one with lower-level nodes coming from it. These nodes are called *child nodes* to that parent node. Nodes that terminate in the hierarchy are called *leaf nodes*. Leaf nodes are terminal nodes in that they have no children coming from them. For instance, System 1 (in Figure 4.40) is a parent node with M children nodes that are also leaf nodes. These nodes are subsystem 11 through subsystem $1M$.

From a TPM perspective, an SoS hierarchy should be decomposed to the level at which the contributions of individual TPMs can be directly evaluated and a TRI computed. The following offers a way to compute a TRI for a system-of-systems. This is followed by an illustrative case discussion.

*Chairman of the Joint Chiefs of Staff Manual (CJCSM 3170.01, 24 June 2003).

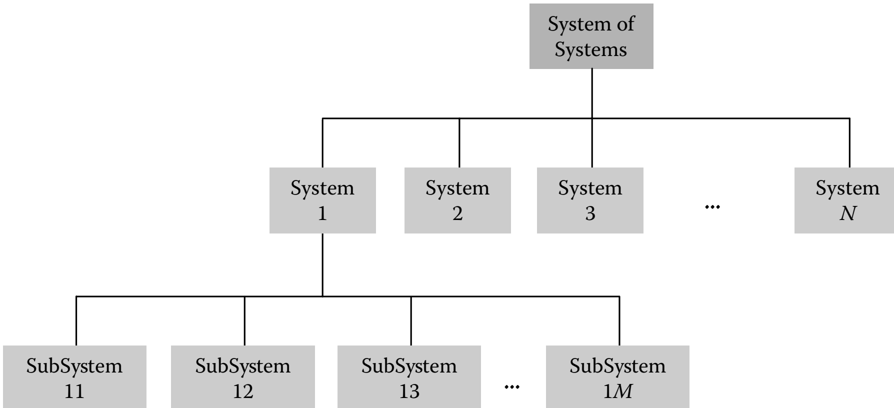


Figure 4.40: A system-of systems hierarchical structure.

Computing TRI

The TRI of a system-of-systems is computed as a logical combination of the TRIs across the leaf nodes of the hierarchy. Specifically, a $\text{TRI}_{ti,\text{All}}$ is computed for each leaf node x, in the same way presented in Equation 4.72. Denote this value as $\text{TRI}_{ti,x}$ where the subscript x is to represent the set of all TPMs that are applicable to the leaf node x. Next, the $\text{TRI}_{ti,x}$ at all leaf nodes are combined to derive the $\text{TRI}_{ti,\text{ SoS}}$ at the system-of-systems level of the hierarchy. To describe this process, we further generalize the notation $\text{TRI}_{ti,x}$ to denote the TRI value for any node x, leaf or parent, in the SoS hierarchy and the subscript x now represents all the TPMs that are applicable to the node x, directly (as for a leaf node) or indirectly (as for a parent node).

Combining TRI for a parent node from its children (leaf or lower-level parent nodes) can be done according to the following rule. The overall TRI for a parent node k with M children (nodes $k1, \ldots, kM$) at time ti can be written as follows:

$$\text{TRI}_{ti,k} = (\text{w}_{k0}\text{TRI}_{ti,k0} + \text{w}_{k1}\text{TRI}_{ti,k1} + \cdots + \text{w}_{kM}\text{TRI}_{ti,kM})/ \\ (\text{w}_{k0} + \text{w}_{k1} + \cdots + \text{w}_{kM}) \tag{4.73}$$

where node $k0$ is an added child to the parent node k to represent the set of TPMs that are applicable across multiple or all original children of parent node k.

Starting at the lowest level of an SoS hierarchy, Equation 4.73 can be used to compute the TRI for all parent nodes — as appropriate to the structure of a given SoS hierarchy. Thus, the overall TRI for an SoS hierarchy composed of N systems (i.e., with nodes 1, . . ., N as children to the topmost node of the hierarchy) is

$$\text{TRI}_{ti,\,\text{SoS}} = (\text{w}_0\text{TRI}_{ti,0} + \text{w}_1\text{TRI}_{ti,1} + \cdots + \text{w}_N\text{TRI}_{ti,\,N})/(\text{w}_0 + \text{w}_1 + \cdots + \text{w}_N) \tag{4.74}$$

where system 0 is an added child to the top SoS node to represent the set of TPMs that are applicable across multiple or all systems listed as children under the top node.

In Figure 4.40, suppose the system 1 parent node ($k = 1$) has just $M = 3$ subsystems (subsystems 11, 12, and 13) as its children. Besides the TPMs that are to be measured at each of the subsystems, we assume there is also a set of TPMs that are applicable across multiple or all subsystems (e.g., subsystem-to-subsystem integration or system-level integration). For notational convenience, we use subsystem 10 to denote the collection of such TPMs and use $\text{TRI}_{ti,10}$ to denote the TRI value computed on those TPMs. Then, the overall TRI of system 1 at time ti is as follows:

$$\begin{aligned}\text{TRI}_{ti,1} = {} & (\text{w}_{10}\text{TRI}_{ti,10} + \text{w}_{11}\text{TRI}_{ti,11} + \text{w}_{12}\text{TRI}_{ti,12} + \text{w}_{13}\text{TRI}_{ti,13})/ \\ & (\text{w}_{10} + \text{w}_{11} + \text{w}_{12} + \text{w}_{13})\end{aligned} \tag{4.75}$$

Clearly, if the system 1 parent node's TRI is defined *solely* by its children's TRI values then Equation 4.75 can be simplified with w_{10} set equal to zero.

The above equations apply a weighted average rollup rule for determining the TRI values in the SoS hierarchy. This rule is appropriate for a parent node when its children's performance levels are considered additive in measuring the parent node's performance level. This implies, with their assigned weights, all children's risk levels directly add to the parent node's risk level. This is a common rule to use in the rollup of TRI values; however, other rules may also be defined and applied accordingly. Such rules are discussed further in reference 8.

Case Discussion 4.3 Suppose we have a defense system made up of the SoS and subsystems shown in Figure 4.41. Suppose Defense System 1 is made up of five subsystems. These are an *Engagement Subsystem*, a *Sensor Subsystem*, a *Tracker Subsystem*, a *Sensor Manager Subsystem*, and a *Communications* (*Comms*) *Subsystem*.

For each of these five subsystems, suppose their TPMs are defined as shown in Figure 4.41. Notice these are a mix of Category A and Category B TPMs. From the data in Table 4.24 determine the TRI for Defense System 1.

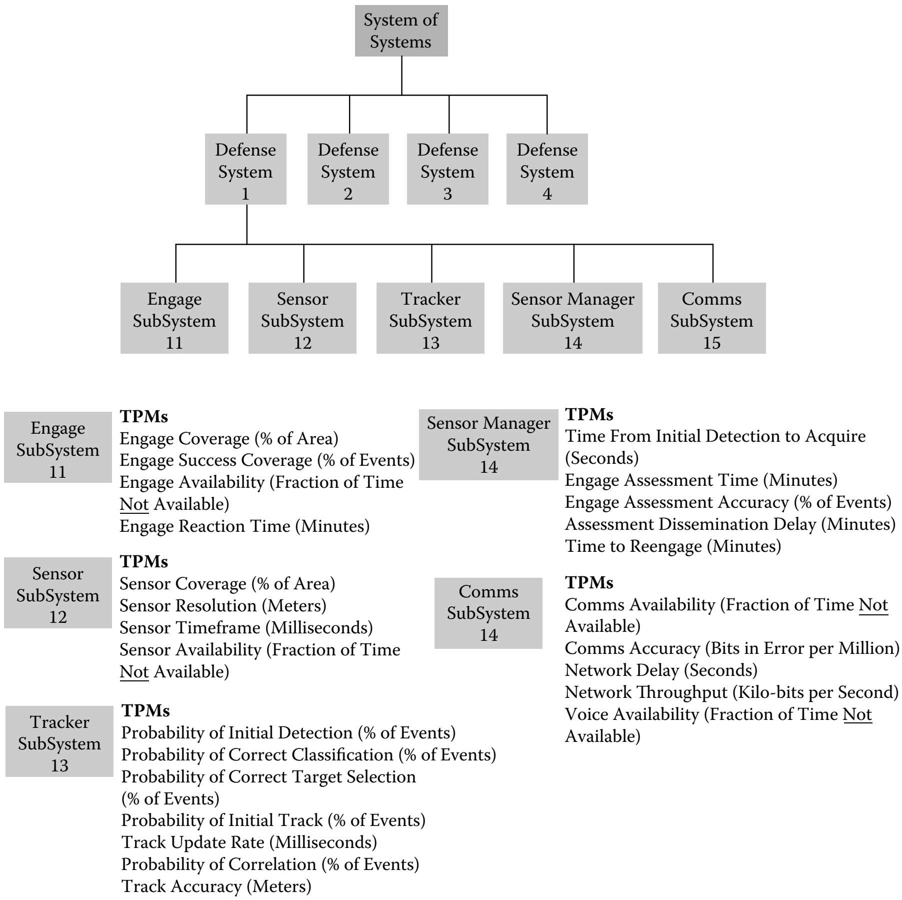


Figure 4.41: An Illustrative defense system: a system-of-systems.

From the data in Table 4.24 we can derive the following *Engage Subsystem* TRI as summarized in Table 4.25. Similar computations are done for the four remaining subsystems. This is left for the reader. Table 4.26 shows a summary of these computations.

Given the individual subsystem TRIs in Table 4.26 we next compute the overall System 1 TRI. Let's also assume the importance weight of each Defense System 1 subsystem is one. Also, suppose there is a common collection of TPMs that cut across the subsystems of Defense System 1 and that the TRI for these TPMs was computed to be equal to 0.350. Given these additional inputs, the overall TRI for

TABLE 4.24: Defense System 1 TPM Data

Individual System 1 Subsystems Measurement Date t1	**Threshold**	**Raw Value**
Engage Subsystem TPMs		
Cat B: Engage Coverage (% of Area)	100.000	80.000
Cat B: Engage Success Coverage (% of Events)	99.000	85.000
Cat A: Engage Availability (Fraction of Time Not Available)	0.00010	0.010
Cat A: Engage Reaction Time (Minutes)	2.000	5.000
Sensor Subsystem TPMs		
Cat B: Sensor Coverage (% of Area)	100.000	75.000
Cat A: Sensor Resolution (Meters)	1.000	4.000
Cat B: Sensor Timeframe (Milliseconds)	5.000	2.000
Cat A: Sensor Availability (Fraction of Time Not Available)	0.00010	0.010
Tracker Subsystem TPMs		
Cat B: Probability of Initial Detection (% of Events)	98.000	85.000
Cat B: Probability of Correct Classification (% of Events)	95.000	80.000
Cat B: Probability of Correct Target Selection (% of Events)	90.000	75.000
Cat B: Probability of Initial Track (% of Events)	90.000	75.000
Cat A: Track Update Rate (Milliseconds)	10.000	20.000
Cat B: Probability of Correlation (% of Events)	80.000	60.000
Cat A: Track Accuracy (Meters)	1.000	4.000
Sensor Manager Subsystem TPMs		
Cat A: Time From Initial Detection to Acquire (Seconds)	5.000	10.000
Cat A: Engage Assessment Time (Minutes)	1.000	4.000
Cat B: Engage Assessment Accuracy (% of Events)	90.000	80.000
Cat A: Assessment Dissemination Delay (Minutes)	1.000	2.000
Cat A: Time to Reengage (Minutes)	0.500	1.000
Comms Subsystem TPMs		
Cat A: Comms Availability (Fraction of Time Not Available)	0.00010	0.010
Cat A: Comms Accuracy (Bits in Error per Million)	1.000	4.000
Cat A: Network Delay (Seconds)	10.000	20.000
Cat B: Network Throughput (Kilo-bits per Second)	2.500	1.500
Cat A: Voice Availability (Fraction of Time Not Available)	0.00010	0.010

TABLE 4.25: TRI Computations for Engage Subsystem 11

Cat A TPM	$V_{thres, A}$	**V(ti,** A)	**Eq. 4.62** **v(ti,** A)	**Eq. 4.66** **u(ti,** A)	**wt**
Cat A: Engage Availability (Fraction of time not available)	0.00010	0.010	100.000	0.010	1.000
Cat A: Engage Reaction Time (mins)	2.000	5.000	2.500	0.400	1.000
Cat B TPM	$V_{thres, B}$	**V(ti,** B)	**Eqt 4.63** **v(ti,** B)		**wt**
Cat B: Engage Coverage (% of Area)	100.000	80.000	0.800		1.000
Cat B: Engage Success Coverage (% of Events)	99.000	85.000	0.859		1.000
TRI Computations					
TRI* (t1, A) =		0.795		Eq. 4.70	
TRI (t1, B) =		0.171		Eq. 4.71	
TRI (t1, All) = TRI (t1,11) =		**0.483**		Eq. 4.72	

Defense System 1 is computed, from Equation 4.75, as follows:

$$\mathrm{TRI}_{ti,1} = (\mathrm{w}_{10}\mathrm{TRI}_{ti,10} + \mathrm{w}_{11}\mathrm{TRI}_{ti,11} + \mathrm{w}_{12}\mathrm{TRI}_{ti,12} + \mathrm{w}_{13}\mathrm{TRI}_{ti,13} + \mathrm{w}_{14}\mathrm{TRI}_{ti,14} + \mathrm{w}_{15}\mathrm{TRI}_{ti,15})/(\mathrm{w}_{10} + \mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14} + \mathrm{w}_{15}) \qquad (4.76)$$

$$\mathrm{TRI}_{ti,1} = (\mathrm{w}_{10}(0.350) + \mathrm{w}_{11}(0.483) + \mathrm{w}_{12}(0.648) + \mathrm{w}_{13}(0.304) + \mathrm{w}_{14}(0.473) + \mathrm{w}_{15}(0.726))/(\mathrm{w}_{10} + \mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14} + \mathrm{w}_{15}) \qquad (4.77)$$

$$\mathrm{TRI}_{ti,1} = ((0.350) + (0.483) + (0.648) + (0.304) + (0.473) + (0.726))/6 \qquad (4.78)$$

From which we have

$$\mathrm{TRI}_{ti,1} = 0.497$$

Thus, the TRI for Defense System 1 is 0.497.

TABLE 4.26: Summary TRI Computations for All System 1 Subsystems

Engage Subsystem 11	0.483
TRI (t1, All) = TRI (t1,11)	
Sensor Subsystem 12	0.648
TRI (t1, All) = TRI (t1,12)	
Tracker Subsystem 13	0.304
TRI (t1, All) = TRI (t1,13)	
Sensor Mgt Subsystem 14	0.473
TRI (t1, All) = TRI (t1,14)	
Comms Subsystem 15	0.726
TRI (t1, All) = TRI (t1,15)	

Computing the System-of-Systems TRI

Suppose, at measurement date $t1$, TRI values for Defense System 2, 3, and 4 have been computed and equal 0.60, 0.65, and 0.70, respectively. Suppose the system-to-system TRI was computed to be 0.30. From these data and the results from Case Discussion 4.3, the overall SoS TRI at measurement date t1 is

$$\mathrm{TRI}_{t1,\,\mathrm{SoS}} = (\mathrm{w}_0\mathrm{TRI}_{t1,0} + \mathrm{w}_1\mathrm{TRI}_{t1,1} + \cdots + w_4\mathrm{TRI}_{t1,4})/ (\mathrm{w}_0 + \mathrm{w}_1 + \cdots + \mathrm{w}_4) = 0.549$$

where the above assumed that each Defense System has equal importance weight to the system-of-systems.

Color Determinations

Since the TRI metric is bounded between zero and one it is convenient to express the TRI as a color in addition to its computed value. Figure 4.42 posits a color scheme that can be applied to each node of the SoS hierarchy. From the preceding computations, the SoS hierarchy in this section could be colored according to the picture in Figure 4.42. Here, the magnitude of the TRI can be interpreted as revealing the "strength" of the color, as shown in Figure 4.43.

For instance, the *Sensor Subsystem* is "strongly" orange due to its TRI being close to the red color boundary. The *Tracker Subsystem* is "strongly" yellow. It has a TRI close to the yellow-orange boundary.

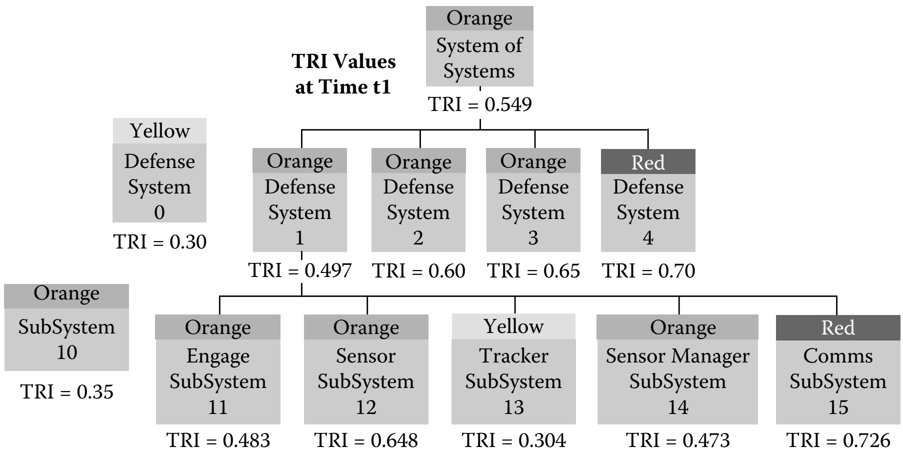


Figure 4.42: Illustrative TRI values and colors.

The TRI color-score mapping provides a logical scale and a convenient way to rollup TRI values and colors across an SoS hierarchy. It provides management a quick visual communication of the TRI value and the overall performance risk index of the system-of-systems.

Finally, research has been published on measuring reductions in performance risk as a function of a system's design maturity and the *level* of performance *valued* by users. Known as the *Risk-Value Method*, this approach integrates utility or value function constructs into traditional performance measurement formalisms (e.g., TPMs) to capture user strength of preferences for improvements in system performance levels. Probability distributions are used to capture uncertainties in performance outcomes as a system's design and engineering matures over time. The reader is directed to reference 9 for a further discussion of the *Risk Value Method*.

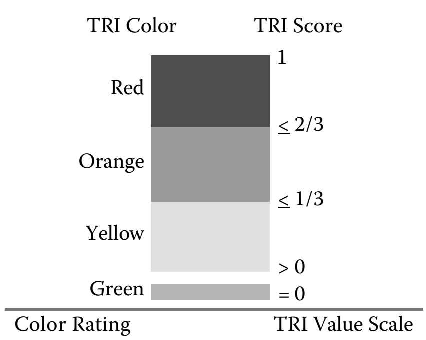


Figure 4.43: A possible TRI color scheme.

The following presents a discussion of risk management in the context of engineering enterprise systems. Enterprise systems engineering is an emerging discipline. It encompasses and extends "traditional" systems engineering to create and evolve "webs" of systems that deliver capabilities via services, data, and applications through a rich network of information and communications technologies. Enterprise environments (such as the Internet) offer users ubiquitous cross-boundary access to a wide-variety of services, applications, and information repositories.

Today, we're in the early stage of understanding how systems engineering, management, and social science methods weave together to create systems that "live" and "evolve" in enterprise environments. The next section discusses some of these understandings, specifically as they pertain to risk management. The analytical practices discussed will themselves evolve as the community gains experience and knowledge about engineering in the enterprise problem space.

4.6 Risk Management for Engineering Enterprise Systems

Engineering today's systems is a sophisticated, complex, and resource-intensive undertaking. Increasingly, systems are being engineered by bringing together many separate systems which, as a whole, provide an overall capability otherwise not possible. Many systems no longer physically exist within clearly defined boundaries; rather, systems are more and more geographically and spatially distributed and interconnected through a rich and sophisticated set of networks and communications technologies.

These large-scale enterprise systems operate to satisfy large, and dynamically changing, user populations, stakeholders, and communities of interest. It is no longer enough to find just technology solutions to the engineering of these systems. Solutions must be adaptable to change in the enterprise, balanced with respect to expected capability outcomes and performance, while also considering the social, political, and economic constraints within which they'll operate and change over time.

In an enterprise context, risk management is envisioned as an integration of people, processes, and tools that together ensure the early identification and resolution of risks. The goal is to provide decision-makers with an enterprise-wide understanding of risks, their potential consequences, interdependencies, and rippling effects within and beyond enterprise "boundaries." Ultimately, risk management aims to establish and maintain a *holistic* view of risks across the enterprise, so capabilities and performance objectives are achieved via risk-informed resource and investment decisions.

4.6.1 The Enterprise Problem Space

Mentioned earlier, today's systems are continually increasing in scale and complexity. Today, more and more defense systems, transportation systems, financial systems, health and human services systems network ubiquitously across boundaries and seamlessly interface with users, information repositories, applications, and services. These systems can be considered, in one sense, an enterprise of people, processes, technologies, and organizations.

A distinguishing feature of "*enterprise*" systems is not only their technologies but the *way* users interface with them and each other. New challenges are present in how to design and engineer these systems, and their interfaces, from human, social, political, and managerial dimensions [10]. To address these challenges the engineering and social sciences are joining together in ways not previously seen, when planning and evolving the design, development, and operation of these large-scale and highly networked systems.

The following discusses the enterprise problem space and systems thinking within that space, and sets a context for how engineering theory and practice might be considered. The materials that follow derive from a perspectives paper on enterprise engineering, written by George Rebovich, Jr. of The MITRE Corporation* [11].

The Enterprise

In a broad context, an enterprise is an entity comprising interdependent resources that interact with each other and their environment to achieve goals**. A way to view an enterprise is illustrated in Figure 4.44. Here, resources include people, processes, organizations, technologies, and funding. Interactions include

*Permission has been granted to excerpt materials from the paper "Enterprise Systems Engineering Theory and Practice, Volume 2: Systems Thinking for the Enterprise: New and Emerging Perspectives," authored by Rebovich, George, Jr., MP 050000043, November 2005. © 2005 The MITRE Corporation, All Rights Reserved.

**This definition is similar to that of *Enterprise* in the *Net Centric Implementation Framework*, v1.0.0, 17 December 2004, Netcentric Enterprise Solutions for Interoperability (NESI).

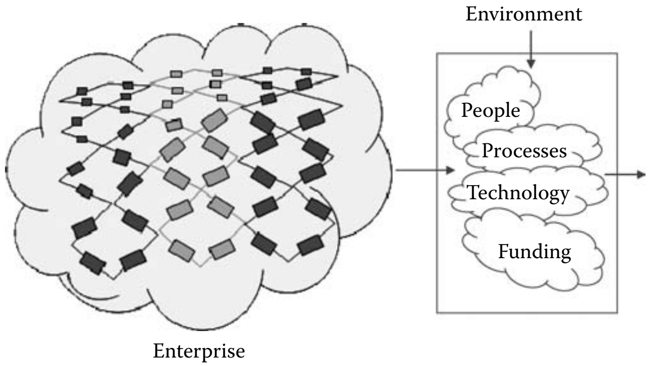


Figure 4.44: An enterprise and its environment. [11]

coordinating functions or operations, exchanging data or information, accessing applications or services.

Historically, systems engineering has focused on the technologies that have enabled the development of the piece parts — the systems and subsystems embedded in the enterprise. Modern systems thinkers like Gharajedaghi [12] are increasingly taking a holistic view of an enterprise. Here, an enterprise can be characterized as the following:

- A multiminded sociocultural entity composed of a voluntary association of members who can choose their goals and means
- An entity whose members share values embedded in a (largely common) culture
- Having the attributes of a purposeful entity
- An entity whose performance improves through alignment of purposes across its multiple levels

There is a nested nature to many enterprises. At every level, except at the very top and bottom, an enterprise itself is part of a larger enterprise and contains sub-enterprises, each with its own people, processes, technologies, funding, and other resources. Nesting within an enterprise can be illustrated by a set of US Air Force programs shown in Figure 4.45. Here, the family of Airborne Early Warning and Control (AEW&C) systems is an enterprise nested in the Command and Control (C2) Constellation enterprise, which is nested in the Air Force C2 enterprise.

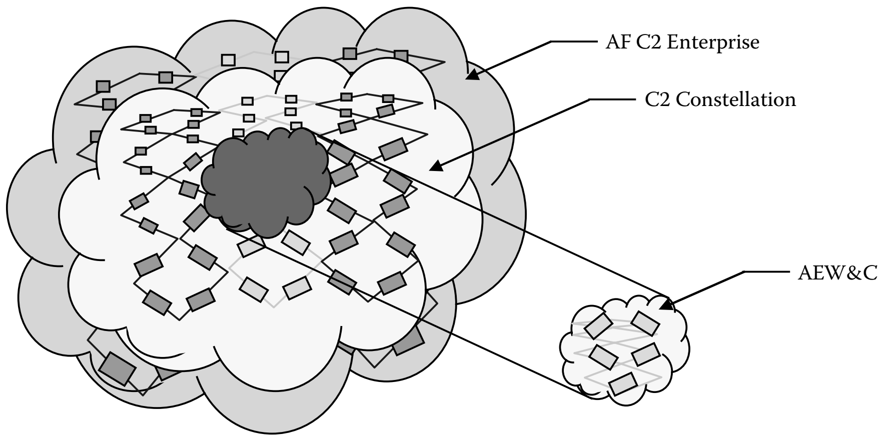


Figure 4.45: Nested nature of enterprises. [11]

Alignment of purposes across the levels of the enterprise can improve overall enterprise performance. The sub-enterprises contribute to the outcomes or goals of the containing enterprise. This view has profound implications for how systems engineers must think about their activities — that they are inexorably linked to the enterprise and its operations as a whole.

For example, at the AEW&C system program level, the view must be that an AEW&C system builds an air picture that serves the higher goal of achieving situation awareness within the C2 Constellation. This requires the AEW&C systems engineer to ask (and answer) how the AEW&C piece parts being developed serve situation awareness in the C2 Constellation *in addition* to how they serve the AEW&C system specification.

At the next level, the view must be that the C2 Constellation develops integrated capabilities to serve the higher goal of providing netcentric C2 for the Air Force C2 enterprise. The implication is that the systems engineer must address how the C2 Constellation piece parts serve the Air Force C2 enterprise, in addition to how they serve the C2 Constellation.

At the highest level, in this example, the view must be that the Air Force C2 enterprise develops Air Force netcentric capabilities to serve the higher goal of providing netcentric C2 for the Joint/Coalition C2 enterprise. The implication is that the systems engineer must address how the Air Force C2 Enterprise piece parts serve joint and coalition netcentric C2 in addition to how they serve the Air Force C2.

This discussion leads to an operational definition of enterprise viewed from the perspective of an individual (system engineer or other participant) or team in the enterprise. It aims to answer the question, "What is my (our) enterprise?" The enterprise, then, can be viewed as a set of interdependent elements (systems and resources) that a participating actor or actors either control or influence.

This definition of enterprise is a virtual construct that depends on the make-up, authority, and roles of the participating actors in a community of interest. For example, the program team of a system managed by one organization may have virtual control of most engineering decisions being made on the system's day-to-day development activities. If the system is required to be compliant with technical standards developed by an external agency, the program team may have representation on the standards team but that representation is one voice of many and so the standard is a program element or variable the program team can influence but not control. The U.S. Federal Acquisition Regulation requirements, which apply to virtually all U.S. government acquisitions, are elements or variables that apply to our example program but, since they are beyond the control of the team, they are part of the program's environment.

The implication is that all actors or teams in an enterprise setting should know "their" enterprise and be aware of which enterprise elements or variables they control, which they influence, and which they neither control nor influence. In general, environmental elements or factors cannot be controlled or influenced. But the individual or project team may very well need to be aware of and understand the implications of such environmental factors.

Systems engineering has always been about asking good questions, answering them, and following their implications. The following is a series of questions that can help an engineer harness the complexity of a particular system or enterprise.

- What is my enterprise? What elements of it do I control? What elements do I influence? What are the elements of my environment that I do not control or influence but which influence me?
- How can a balance be achieved between optimizing at the system level with enabling the broader enterprise, particularly if it comes at the expense of the smaller system?
- How can different perspectives be combined into one view to enable alignment of purposes across the enterprise?
- Would a change in performance at the subsystem level result in a change at the enterprise level? If so, how? Is it important? How would a new

enterprise-level requirement be met and how would it influence systems below it?

- How can complementary relations in opposing tendencies be viewed to create feasible wholes with seemingly unfeasible parts? How can they be viewed as being separate, mutually interdependent dimensions that can interact and be integrated into an "and" relationship?
- Are dependencies among variables in a system or enterprise such that the ability to make progress in one variable occurs at the expense of others? How can dependencies among variables within an enterprise be identified, monitored, managed accordingly?

Finally, this discussion concludes with a lexicon being developed within the systems and engineering communities as a way to drive toward a common understanding of words such as *system*, *complexity*, and *enterprise*. This lexicon was researched and established by Brian E. White of The MITRE Corporation [13]. The following are excerpts from this work.*

What's in a word? Terminology is crucial to understanding terms like *system*, *complexity*, *enterprise*, *systems engineering*, *enterprise systems engineering*. Although many of these words are in use today, this lexicon proposes definitions upon which the systems and engineering communities might, in time, converge to a consensus or common understanding.

System: *An interacting mix of elements forming an intended whole greater than the sum of its parts*. These elements may include people, cultures, organizations, policies, services, techniques, technologies, information/data, facilities, products, procedures, processes, and other human-made (or natural) entities. The whole is sufficiently cohesive to have an identity distinct from its environment.

System-of-Systems (*SoS*)*: A collection of systems that functions to achieve a purpose not generally achievable by the individual systems acting independently*. Each system can operate independently and is managed primarily to accomplish its own separate purpose. A system-of-systems can be geographically distributed and can exhibit evolutionary development and/or emergent behaviors.

*Permission has been granted to excerpt materials from the paper "Fostering Intra-Organizational Communication of Enterprise Systems Engineering Practices," authored by White, Brian, E., October 2006.

Complex System: An open system with continually cooperating and competing elements. This type of system continually evolves and changes its behavior (often in unexpected ways) according to its own condition and its external environment. Changes between states of order and chaotic flux are possible. Relationships among its elements are imperfectly known and are difficult to describe, understand, predict, manage, control, design, or change.

Enterprise: A complex system in a shared human endeavor that can exhibit relatively stable equilibriums or behaviors (homeostasis) among many interdependent component systems. An enterprise may be embedded in a more inclusive complex system. External dependencies may impose environmental, political, legal, operational, economic, legacy, technical or other constraints. An enterprise usually includes an agreed-to (or defined) mission with set goals, objectives, and outcomes.

Engineering: Methodically conceiving and implementing viable solutions to existing problems.

Enterprise Engineering: Application of engineering efforts to an enterprise with emphasis on enhancing capabilities of the whole, while attempting to understand the relationships and interactive effects among the components of the enterprise and with its environment.

Systems Engineering: An iterative and interdisciplinary management and development process that defines and transforms requirements into an operational system. Typically, this process involves environmental, economic, political, social, and other non-technological aspects. Activities include conceiving, researching, architecting, designing, developing, fabricating, producing, integrating, testing, deploying, operating, sustaining, and retiring system elements. The customer (or user) of the system usually states the initial requirements. Systems engineering can then be applied to further define, refine, and evolve these requirements in near and long-term outlooks.

Traditional Systems Engineering (TSE): Systems engineering but with limited attention to the non-technological and/or complex system aspects of the system. In TSE, there is emphasis on the process of selecting and synthesizing the application of the appropriate scientific and technical knowledge to translate system requirements into a system design. Here, it is normally assumed and assured that the behavior of the system is completely predictable. Traditional engineering (not just TSE) is typically directed at the removal of unwanted system or performance behaviors.

Enterprise Systems Engineering (ESE): A regimen for engineering "successful" enterprises. ESE is systems engineering with an emphasis on a body of knowledge, tenets, principles, and precepts having to do with the analysis, design, implementation, operation, and performance of an enterprise. Rather than focusing on parts of the enterprise, the enterprise systems engineer concentrates on the enterprise as a whole — and how its design, as applied, interacts with its environment. Thus, ESE avoids potentially detrimental aspects of TSE by focusing on how all its parts interact. This includes how these parts interact with the outside environment.

4.6.2 Enterprise Risk Management: A Capabilities-Based Approach

What events threaten the delivery of capabilities needed to successfully advance enterprise goals and mission outcomes? If these events occur, how serious are their impacts? How can the progress of management plans, aimed at minimizing their impacts, be monitored? How can risk be considered in resource planning and investment decision-making? Questions such as these arise when planning, executing, and managing the engineering of large-scale enterprise-wide systems. Addressing these questions involves not only engineering and technology dimensions but human–social-system interactions as well.

Enterprise risk management differs from "traditional" systems engineering risk management in the expanse of the consequence space within which risks affect enterprise goals, mission outcomes, or capabilities. In a "traditional" system, the consequence space is usually focused on the extent risks negatively affect the system's cost, schedule, and technical performance. Enterprise risk management necessitates broadening the scope of this space. Identifying and evaluating higher-level effects (or consequences) are critical considerations in decisions on where to allocate resources to manage enterprise risks.

A Capability Portfolio View

One way management plans for engineering an enterprise is to create capability portfolios of technology programs and initiatives that, when synchronized, will deliver time-phased capabilities that advance enterprise goals and mission outcomes. Thus, a capability *portfolio* is *a time dynamic organizing construct* to deliver capabilities across specified epochs.

Creating capability portfolios is a complex management and engineering analysis activity. In the systems engineering community, there is a large body of literature

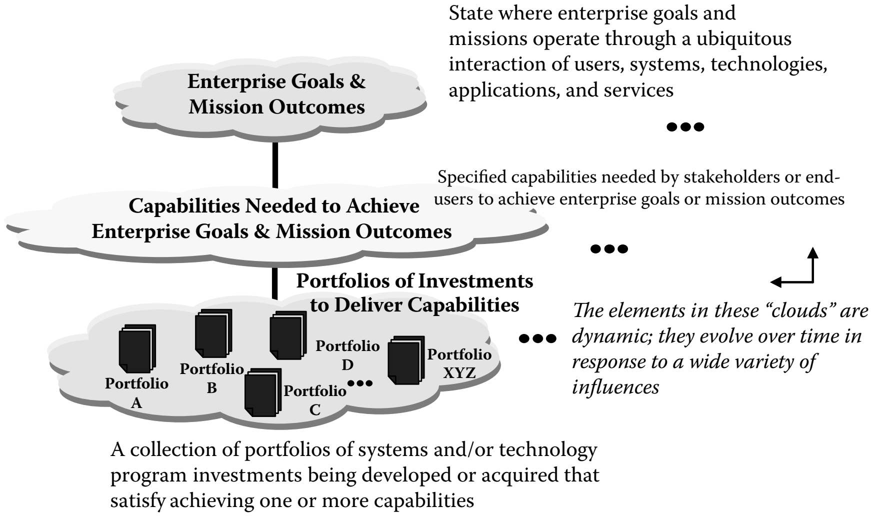


Figure 4.46: An enterprise and its capability portfolios.

on *portfolio analysis and investment decision management* applied to the acquisition of today's advanced systems. This topic, however, is outside the scope of this book. Instead, the following is focused on applying risk management practices within a "generic" model of capability portfolios, already defined to deliver capabilities to an enterprise. Figure 4.46 presents a view of such a model.

In Figure 4.46, the lowest level is the family of capability portfolios. What does a capability portfolio look like? An example is shown in Figure 4.47. Presented is an inside look at a capability portfolio from a *capability-to-functionality* view. Figure 4.47 derives from a capability portfolio for network operations [14]. This is one among many capability portfolios designed to deliver capabilities to the Department of Defense (DOD) Global Information Grid* [15].

Given the above, a capability portfolio can be represented in a hierarchical structure. At the top is the capability portfolio itself. Consider this the Tier 1 level. The next tier down the hierarchy presents capability areas, such as Network Management, Information Assurance, and so forth. These Tier 2 elements depict the functional domains which characterize the capability portfolio. Tier 3 is the collection of capabilities the portfolio must deliver by a specified epoch (e.g., 2012). Here, a capability can be defined as the *ability to achieve an effect to a standard*

*The Department of Defense (DOD) defines the Global Information Grid (GIG) as a *globally interconnected, end-to-end set of information capabilities, associated processes, and personnel for collecting, processing, storing, disseminating, and managing information* [15].

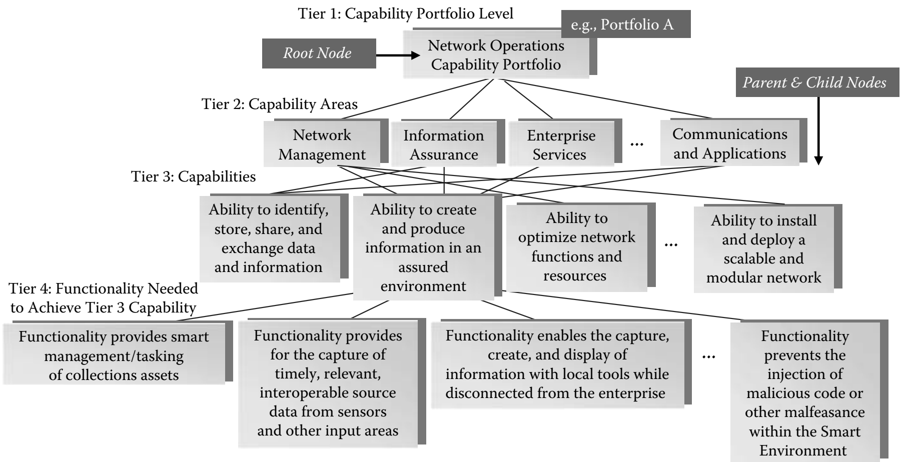


Figure 4.47: A capability portfolio for network operations.* [14]

under specified conditions using multiple combinations of means and ways to perform a set of tasks [14]. Tier 4 is the functionality that must integrate together to achieve capability outcomes.

For example, consider the capability portfolio in Figure 4.47. The Tier 3 capability *Ability to create and produce information in an assured environment* refers to the ability to collect data and transform it into information, while also providing end-to-end protection to assure the availability of information and validating its integrity [14].

Suppose this capability advances toward outcome goals when functionality is delivered that ensures the *Capture of timely, relevant, interoperable source data from sensors and other input areas*. Suppose this functionality contributes to this capability's outcome when the *Time for information change to be posted and/or subscribers notified "< 1 minute"* [14].

Later, we will use this information and show how a hierarchical representation of a capability portfolio can be used as a "modeling" framework within which risks can be assessed and capability portfolio risk measures derived. In preparation for this, we first consider a capability portfolio from a "supplier-provider" context.

*This figure derives, in-part, from *Net-Centric Operational Environment Joint Integrating Concept*, Version 1.0, Joint Chiefs of Staff, 31 October 2005, Joint Staff, Washington, D.C. 20318-6000; reference http://www.dod.mil/cio-nii/docs/netcentric_jic.pdf.

Supplier–Provider Concept

Once a capability portfolio's hierarchy and its elements are "defined" it is managed by a team to ensure its collection of technology programs and technology initiatives combine in ways to deliver one or more capabilities to the enterprise. Thus, one can take a *supplier-provider* view of a capability portfolio. This is illustrated in Figure 4.48.

Here, a capability portfolio can be viewed as the "provider" charged with delivering time-phased capabilities to the enterprise. Technology programs and technology initiatives aligned to, and synchronized with, the capability portfolio "supply" the functionality needed to achieve the provider's capability outcomes.

The supplier-provider view offers a way to examine a capability portfolio from a "risk-perspective." Look again at Figures 4.46, 4.47, and 4.48. Observe that we have enterprise goals and mission outcomes dependent on capability portfolios successfully delivering required capabilities. Next, we have capability portfolios dependent on programs and technologies successfully delivering functionality that enables these capabilities. Thus, major sources of risk originate from the "suppliers" to these capability portfolios.

Supplier risks include unrealistic schedule demands placed on them by portfolio needs or placed *by* suppliers on their vendors. Supplier risks include premature use of technologies, including the deployment of technologies not adequately tested. Dependencies among suppliers can generate a host of risks, especially when a problem with one supplier generates a series of problems with others. Economic conditions can always threaten business stability or the business viability of suppliers and vendors. Unfavorable funding or political influences outside an enterprise can adversely affect its capability portfolios, its suppliers, or the supplier-vendor chains in ways that threaten the realization of enterprise goals and mission outcomes.

The following illustrates an analytical framework within which to structure capability portfolio risk assessments. This framework can be extended to a generalized "logical model" — one where capability portfolio risk assessments combine to measure and trace their integrative effects on engineering the enterprise. Finally, we will discuss the framework and logical model in the context of time-phased capabilities across an incremental capability development approach.

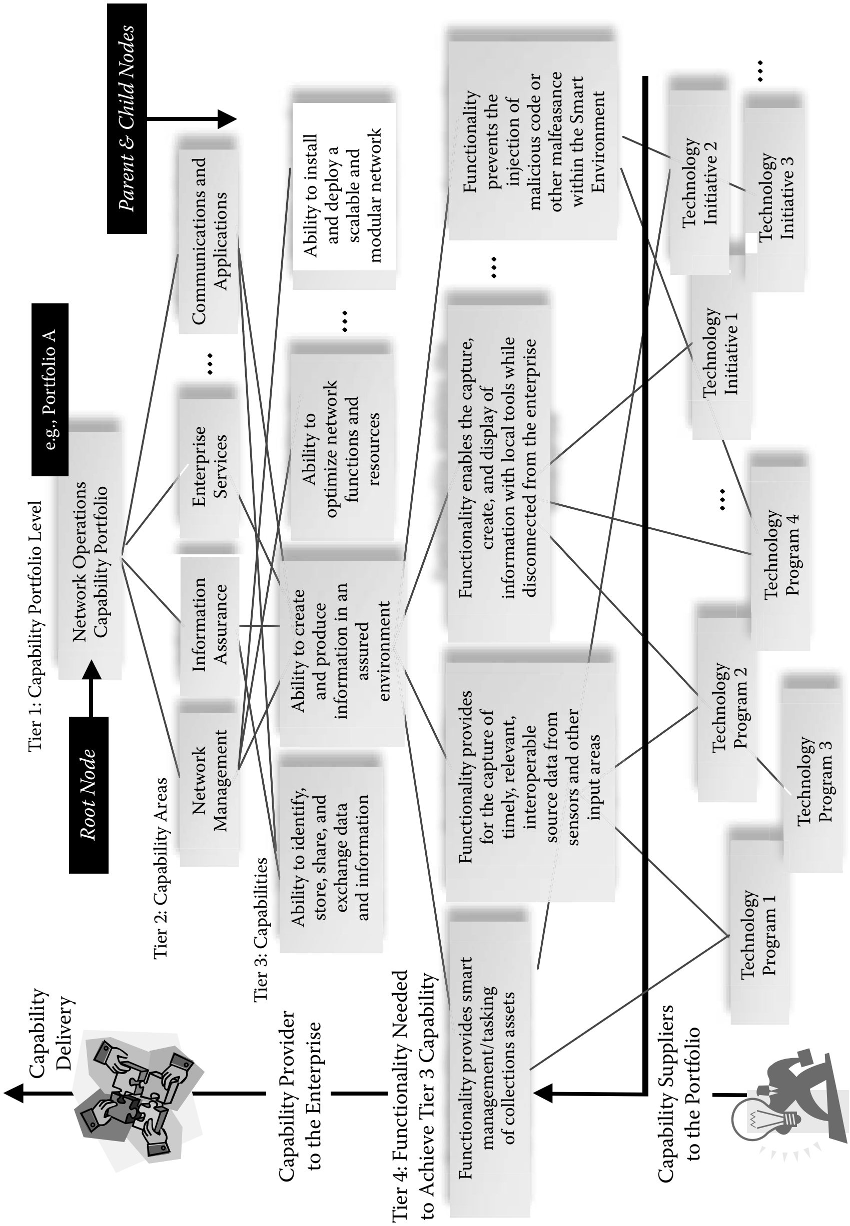


Figure 4.48: A supplier–provider view.

The Capability Portfolio: A Framework for Assessing and Measuring Capability Risk

When a capability portfolio can be represented in a hierarchical structure it offers a "modeling" framework within which risks can be assessed and capability risk measures derived. The following illustrates this idea using the hierarchies in Figure 4.47 and Figure 4.48. In the context of a capability portfolio, we will consider *capability risk as a measure of the chance and the consequence that a planned capability, defined within a portfolio's envelope, will not meet intended outcomes by its scheduled delivery date*.

First, we'll design "algebraic rules" for computing risk measures within a segment of a capability portfolio's hierarchy. Then, we will show how to extend these computations to operate across a capability portfolio's fully specified hierarchy. This will involve a series of rollup calculations. Shown will be risk measure (risk score) computations that originate from leaf nodes, which will then rollup to measure the risks of parent nodes, which will then rollup to measure the risk of the capability portfolio itself (i.e., the root node level).

When a capability portfolio can be represented in the form of a hierarchy, decision-makers can be provided with the trace basis and the event drivers behind all risk measures derived for any node at any level in the hierarchy. From this, management has visibility and supporting rationales for identifying where resources are best allocated to reduce (or eliminate) risk events that threaten the success of the capability portfolio's goals and capability outcome objectives.

Designing the "Algebra" of a Capability Portfolio

In a capability portfolio's hierarchical structure, each element in the hierarchy is referred to as a "node." The top-most node is the *root node*. In Figure 4.48 or Figure 4.49 the root node represents the capability portfolio itself, which, in this case, is the Network Operations Capability Portfolio. A *parent node* is one with lower-level nodes coming from it. These lower-level nodes are called *child nodes* to that parent node. Nodes that terminate in the structure are called *leaf nodes*. Leaf nodes are terminal nodes in that they have no children coming from them.

In the context of a hierarchy, leaf nodes are terminal nodes that originate from supplier nodes. Here, leaf nodes are risk events associated with supplier nodes. Thus, the risk measures (risk scores) of leaf nodes drive the risk measures of

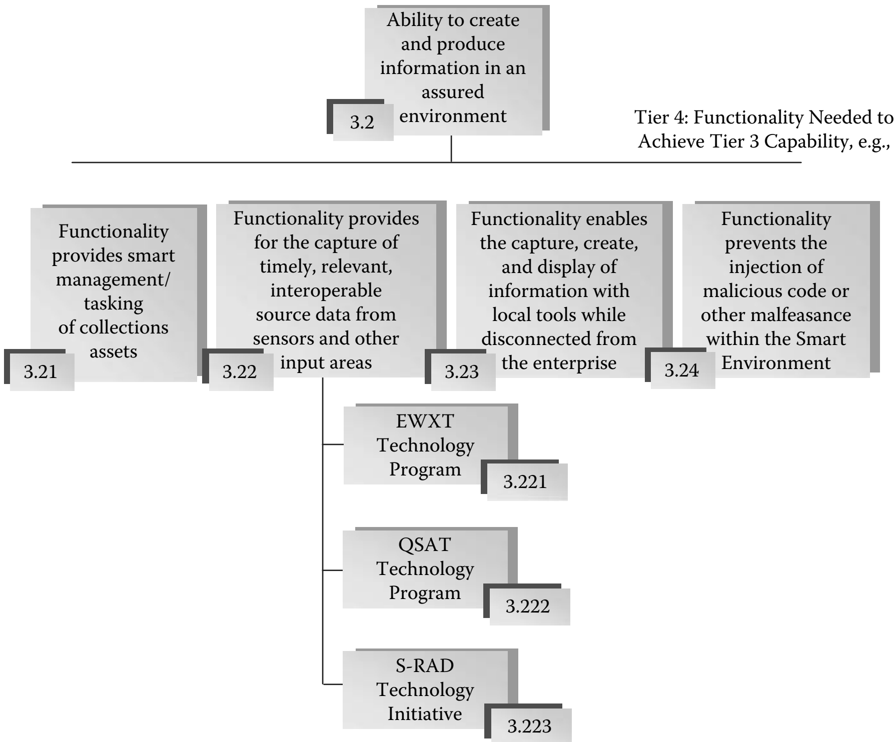


Figure 4.49: A Tier 3 capability from the portfolio in Figure 4.48.

supplier nodes. The risk measures (risk scores) of supplier nodes drive the risk measures of their parent nodes. The risk measures (risk scores) of parent nodes drive the risk measures of their parent nodes, and so forth. Hence, risk measures (risk scores) computed for all nodes originate from risk measures derived for leaf nodes. This *ripple-in-the-pond* effect is reflective of capability portfolio risk management when taking a "supplier-provider" view.

Risks that trace to "suppliers" are a major source of risk to the portfolio's ability to deliver capability to the enterprise. However, it is important to recognize that suppliers are not the only source of risk. Risks external to a capability portfolio's supplier-provider envelope are very real concerns. Risk sources outside this envelope must also be considered when designing and implementing a formal risk management program for a capability portfolio or family of capability portfolios.

Figure 4.49 shows a Tier 3 capability from the portfolio in Figure 4.48. For convenience we've numbered the nodes as shown. Figure 4.49 shows three supplier nodes responsible for contributing to Functionality 3.22 — one of four functions needed for Capability 3.2 to be delivered as planned. Functionality node 3.22 is a parent node to the supplier nodes EWXT, QSAT, and S-RAD.

Two of these supplier nodes are technology programs. One supplier node is a technology initiative. In practice, this distinction can be important. A technology program is often an engineering system acquisition — one characterized by formal contracting, well-defined requirements, and adherence to engineering standards and program management protocols. A technology initiative is often targeted at developing a specific technology for an engineering system or for an appropriate user community. An example might be the development of advanced encryption technology for the information assurance community.

Whether supplier nodes are technology programs or technology initiatives, they exist in a capability portfolio because of their contributions to parent nodes. Seen from the portfolio perspectives in Figure 4.48 and Figure 4.49, functionality nodes are the parent nodes to these supplier nodes. Here, supplier node contributions integrate in ways that enable functionality nodes. Functionality nodes integrate in ways that enable their corresponding capability nodes — capabilities the portfolio is expected to successfully deliver to the enterprise. At the supplier level, we define *contribution* by a supplier node as *that which advances the portfolio's ability to provide capability that meets the needs of the portfolio's consumers*. A supplier's contribution to its parent node (e.g., a functionality node) could be in many forms and include technologies, engineering analyses, or software applications.

At the supplier level, risk events can have adverse consequences on the cost, schedule, or technical performance of the supplier's contribution(s) to its parent node, such as a functionality node in Figure 4.49. Risk events can also negatively affect a supplier's programmatic efforts.

Programmatic efforts refer to technical or program-related work-products as they support the supplier's business, engineering, management, or acquisition practices needed to advance the outcome objectives of the supplier's contribution to its parent node (e.g., a functionality node). Technical or program-related work-products include architecture frameworks, engineering analyses, organizational structures, governance models, and engineering, program, and acquisition management plans.

In addition to the above, supplier nodes can be negatively affected by political risks, budgetary risks, business/economic risks, or supplier/vendor viability risks (where these latter two are "industrial-base" types of risks). These risks not only

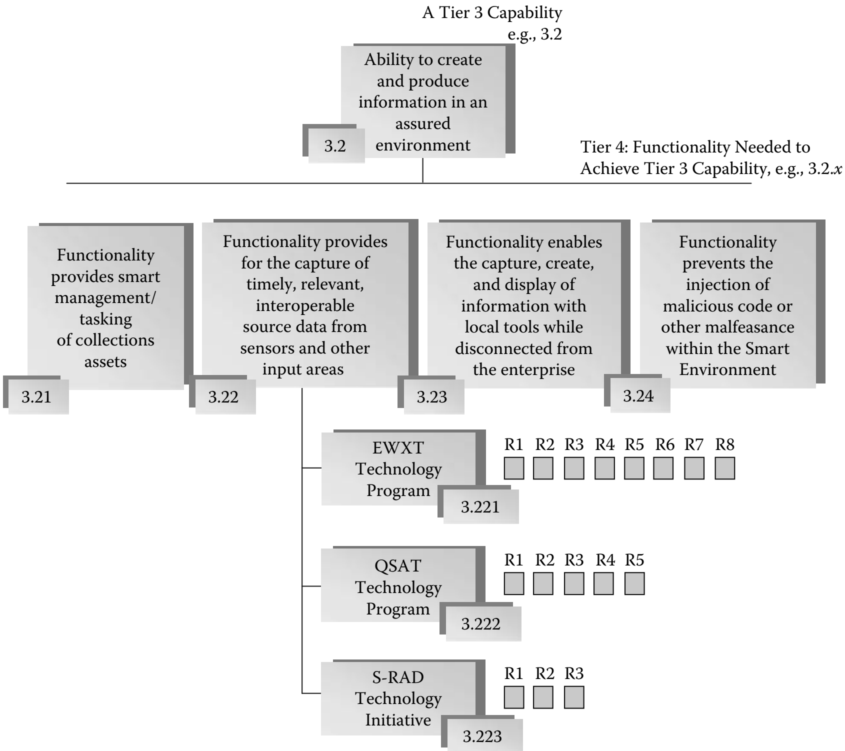


Figure 4.50: Capability 3.2 supplier risk set.

threaten suppliers but they can *directly* threaten functionality or capability nodes at those levels in the capability portfolio's hierarchy. Thus, risk events from a capability portfolio perspective are of multiple types with the potential for multi-consequential impacts on parent nodes located at any level in the hierarchy.

Figure 4.50 shows leaf nodes intended to represent supplier node risk events. These leaf nodes, labeled R1, R2, R3, etc., denote risk events that, if they occur, would negatively affect the supplier node's contribution to its parent node (Functionality 3.22, in this case).

Risks that threaten supplier node contributions to Functionality 3.22 have *"ripple-in-the-pond"* effects on the portfolio's delivery expectations for Capability 3.2. As we'll see, risks that affect Capability 3.2 can have horizontal and vertical effects elsewhere in the portfolio.

Next, we'll look at the EWXT Technology Program. Suppose eight risks have been identified. Denote these by R1, R2, R3, . . . , R8. From a capability portfolio perspective, these are *only* the risk events originating from the EWXT Program that,

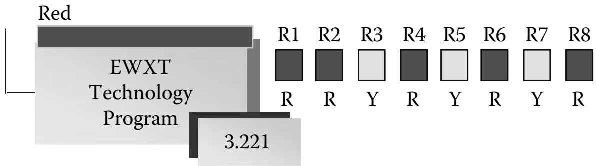


Figure 4.51: EWXT technology program risk set (R = Red, Y = Yellow).

if they occur, have negative consequences on the EWXT Program's *contribution* to Functionality 3.22. In this sense, they may not be all the risk events on the EWXT Program.

In Figure 4.51, each EWXT risk event is given a color. The color reflects a measure of the risk event's severity. In Figure 4.51, each risk event happens to be either Red (R) or Yellow (Y). Suppose the basis for each color derives from a function of each risk event's occurrence probability and its consequence.

Suppose this function is given by Equation 4.79,* where the risk measure (or risk score) of risk event Ri $(i = 1, 2, 3, \ldots, n)$ is defined by

$$Risk\ Score(Ri) = RS_{Ri} = u_1 Prob(Ri) + u_2 V_{Impact}(Ri) \tag{4.79}$$

In Equation 4.79, the first-term is an assessment of the risk event's occurrence probability. The second-term is an assessment of its impact severity, assuming the risk occurs, on the *contribution* the EWXT Program (a supplier node) is making to Functionality 3.22 (its parent node). Both terms can be represented as value functions. In Equation 4.79, suppose these value functions are scaled to produce measures which fall between zero and one-hundred. Table 4.27 offers a constructed scale for the $V_{Impact}(Ri)$ term in Equation 4.79. In Equation 4.79, the coefficients u_1 and u_2 are non-negative weights that sum to one.

Equation 4.79 generates a risk measure (or risk score) for each risk event Ri associated with a supplier node. Next, we will discuss ways to combine these measures into an overall measure of risk for a supplier node. Then, we will discuss ways to combine supplier node risk measures into an overall measure of risk for its parent node (e.g., Functionality 3.22). For this, we introduce the idea of criticality — that is, considering the criticality of a supplier node's contribution to its parent node when measuring that parent node's risk.

*Equation 4.79 is one of many ways to formulate a *Risk Score* measure. The reader is directed to sections 4.3.2 and 4.3.3 for additional approaches to formulating Equation 4.79.

TABLE 4.27: A Constructed Scale: Supplier Node Impacts

Table 4.27 Background Discussion

This table is used to assess a risk event's impact on a supplier node's contribution to its parent node.

Scale Type: In decision analysis, this table is known as a *constructed scale*. Constructed scales are frequently created when natural measurement scales either do not exist or cannot be commonly defined for the problem at hand.

Scale Definitions: The linguistic definitions shown for each scale level derives, in part, from measurement research by D. Meister (see reference). Meister derived sets of linguistic phrases used to indicate an entity's measure of value or goodness in ways that reflect an "ordered-metric". Ordered-metric, in this context, means phrases are at least one-standard deviation apart and have parallel wording or that intervals (levels) between phrases are as nearly equal as possible.

Basis of Assessment (BOA): In practice, all rating assessments **must** be accompanied by a Basis of Assessment (BOA) written such that it (1) clearly and concisely justifies the team's rationale and (2) enables this justification to be objectively reviewed by subject "peers".

Quantified Consequences: If quantified metrics are available that provide context for a risk event's impact(s), then they **must** be included in the BOA justification narratives.

This table provides a *constructed scale* from which an accompanying value function can be developed. Value functions can be developed for each risk "type"; that is, whether it is a cost, schedule, technical performance, or programmatic risk; whether it is a political risk, budgetary risk, economic risk, or business/vendor viability risk, etc. Doing this depends on the level of analytic detail needed or desired by the analysis team. *At a minimum, it is recommended each risk be "type-tagged" so tracking risks in this way can be done as part of the analysis and output summaries to management.*

Ref: *Meister, David (1985). Behavioral Analysis and Measurement Methods, John Wiley & Sons, New York, New York, ISBN 0471896403.*

A Constructed Scale: Supplier Node Impacts

Ordinal Scale/ Level (Score)	Risk Event Impacts on a Supplier Node's Contribution to its Parent Node	Cardinal Interval* Scale/Level (Score)
5	A risk event that, if it occurs, impacts the supplier node to the extent that its contribution to its parent node is severely degraded or compromised. The nature of the risk is such that outcome objectives for the supplier node's contribution are either not met or are ***extremely unacceptable*** (e.g., fall well-below minimum acceptable levels).	e.g., 80 to 100
4	A risk event that, if it occurs, impacts the supplier node to the extent that its contribution to its parent node is marginally below minimum acceptable levels. The nature of the risk is such that outcome objectives for the supplier node's contribution are ***moderately unacceptable.***	e.g., 60 to < 80
3	A risk event that, if it occurs, impacts the supplier node to the extent that its contribution to its parent node falls well-below stated objectives but remains enough above minimum acceptable levels. The nature of the risk is such that outcome objectives for the supplier node's contribution are ***borderline acceptable.***	e.g., 40 to < 60
2	A risk event that, if it occurs, impacts the supplier node to the extent that its contribution to its parent node falls below stated objectives but falls well-above minimum acceptable levels. The nature of the risk is such that outcome objectives for the supplier node's contribution are ***reasonably acceptable.***	e.g., 20 to < 40
1	A risk event that, if it occurs, impacts the supplier node to the extent that its contribution to its parent node is negligibly affected. The nature of the risk is such that outcome objectives for the supplier node's contribution are ***completely acceptable,*** but regular monitoring for change is recommended.	e.g., 0 to < 20

* A linear value function is assumed for illustrative purposes. Other functional forms are possible, such as those discussed in chapter 3 (section 3.2 and 3.4) and in section 4.3.2.

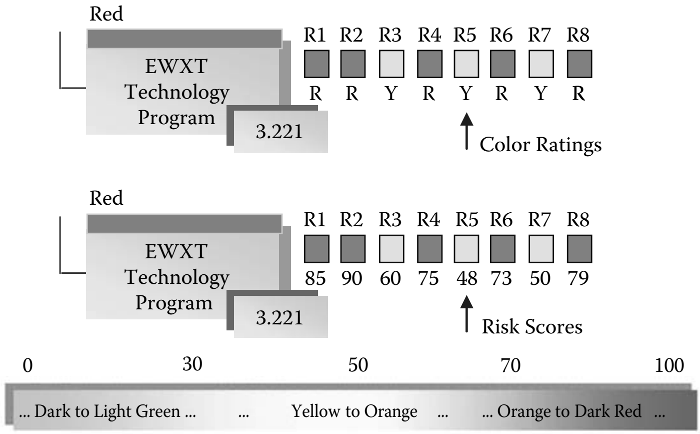


Figure 4.52: Example risk scores for EWXT program risks. (R = Red, Y = Yellow)

Returning to Figure 4.51, Equation 4.79 will produce a risk score for each identified risk event Ri. For convenience, suppose each EWXT risk event's risk score *was already computed* (e.g., by Equation 4.79) and is given in Figure 4.52.

In Figure 4.52, risk event R1 has a risk score of 85; risk event R2 has a risk score of 90; risk event R3 has a risk score of 60 and so forth. Given these eight risk scores for the EWXT Technology Program, what is an overall measure of the risk EWXT poses to Functionality 3.22*? Below is one way to formulate this measure.

Maximum "Max" Average

In section 4.3.3 we introduced the maximum "max" average formulation. From Definition 4.1, the max average of $x_1, x_2, x_3, \ldots, x_n$ where $0 \le x_i \le 100$ (in this case) for all $i = 1, 2, 3, \ldots, n$ is

$$Max\,Ave = \lambda m + (1 - \lambda)\,Average\,\{x_1, x_2, x_3, \ldots, x_n\} \qquad (4.80)$$

where $m = Max\{x_1, x_2, x_3, \ldots, x_n\}$ and λ is a weighting function.

*All risk measures in a portfolio's hierarchical structure reflect the risk situation at a given point in time; thus, they should be regularly reviewed and updated accordingly.

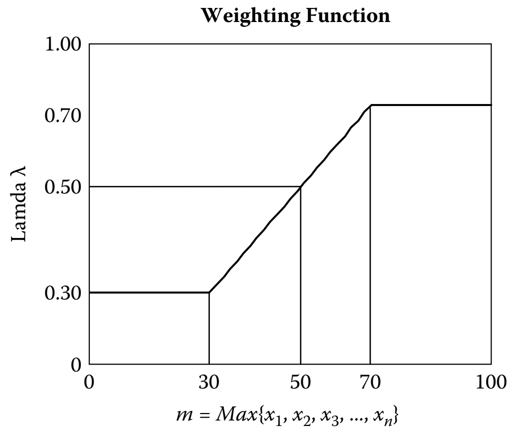


Figure 4.53: An example max average weighting function.

In section 4.3.3, this weighting function was given by Equation 4.14. However, other weighting functions can be formulated.* Suppose the capability portfolio's management decided to use the weighting function in Figure 4.53.

Now, in the context of this problem the x_i's in Equation 4.80 equate to the Ri's (the risk scores) in Figure 4.52. Thus, from Equation 4.80 we have

$$\begin{aligned} \textit{Risk Score}(\text{EWXT}) &= RS_{3.221} \\ &= \lambda(90) + (1-\lambda)\textit{Average}\,\{85, 90, 60, 75, 48, 73, 50, 79\} \end{aligned}$$

where $m = Max\{85, 90, 60, 75, 48, 73, 50, 79\} = 90$. It follows (from Figure 4.53) that $\lambda = 0.70$. From this, we have

$$\textit{Risk Score}(\text{EWXT}) = RS_{3.221} = (0.70)(90) + (0.30)(70) = 84$$

Thus, the EWXT Technology Program (a supplier node) has a high risk score. According to the scale convention in Figure 4.52, EWXT falls in the "RED R" color band.

In summary, the EWXT Technology Program is contributing a high degree of risk toward Functionality 3.22, which threatens Capability 3.2. Furthermore, it can be shown that R1, R2, R4, R6, and R8 are responsible for 93% of the EWXT Program's risk score. These five risk-driving events signal areas in the EWXT Program where increased management focus and risk mitigation planning may

*The shape of the weighting function can have a significant influence on scores generated by the max average rule. In practice, its shape should be designed to model the team's (or decision-maker's) preferences for how much the maximum score should influence the overall score.

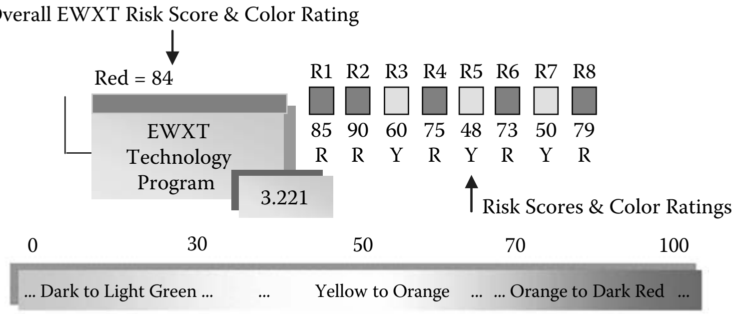


Figure 4.54: Overall EWXT program risk score and color rating. (Max Ave, R = Red, Y = Yellow).

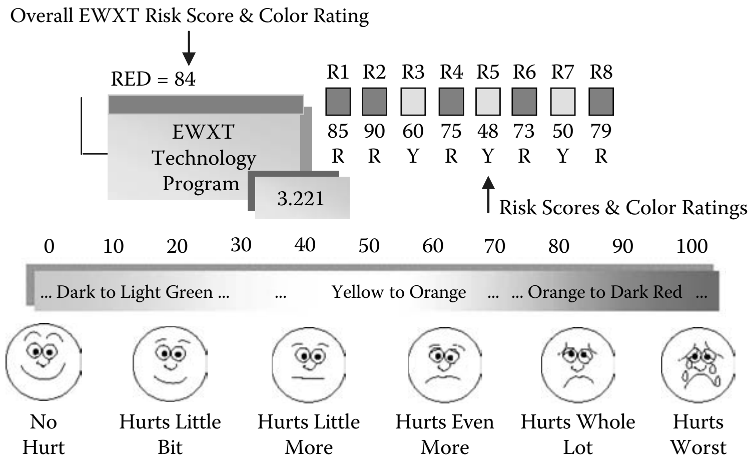


Figure 4.55: Overall EWXT Program Risk Score and Color Rating (with accompanying visual analog scale).

be warranted. Figure 4.54 and Figure 4.55 offer summary views of the EWXT Program risks.

In Figure 4.55, the lower portion presents a feature known as a *visual analog scale*. Visual analog scales are popular protocols in medical communities for assessing pain levels. The example in Figure 4.55 is the well-known Wong-Baker FACES,* a popular visual analog scale.

Visual analog scales can also be used in engineering risk management. They facilitate assessing a risk event's consequence or impact severity (i.e., "pain")

*From Hockenberry, M. J., Wilson, D., Winkelstein M. L.: *Wong's Essentials of Pediatric Nursing*, ed. 7, St. Louis, 2005, p.1259. Used with permission. Copyright, Mosby.

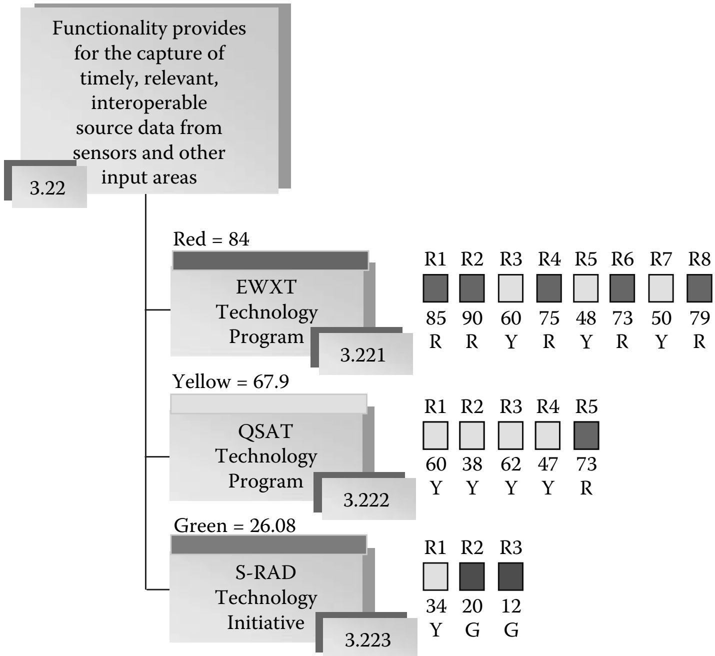


Figure 4.56: Supplier node risk measures to functionality 3.22.

when an objective basis for such an assessment is difficult, or not possible, to be explicitly made.

Measuring "Up": How Supplier Risks Affect Functionality

The preceding discussion presents one way to derive a risk measure (i.e., the risk score) of the EWXT Program, as a function of its eight identified risk events. However, EWXT is just one of three supplier nodes to Functionality 3.22. What about the other supplier nodes? How might their risk measures combine into an overall measure of risk to Functionality 3.22? What ripple effects do supplier risks have on all dependent higher-level nodes in the capability portfolio's hierarchy? The following will address these and related questions.

Suppose risk measures for the other two supplier nodes to Functionality 3.22 are shown in Figure 4.56. These are the QSAT Program and the S-RAD Technology Initiative. Suppose their risk measures were also derived by the max average rule given by Equation 4.80. From this, how can we combine the risk measures from all three supplier nodes in Figure 4.56 into an overall measure of risk to Functionality 3.22? One way is to apply a *variation* of the max average rule to the set of risk scores derived for the supplier nodes. We'll call this variation the "critical" average, which is defined below.

Definition 4.2 *Critical Average:* Suppose a parent node has n child nodes and $\{x_1, x_2, x_3, \ldots, x_n\}$ is the set of scores of these child nodes. If A is a *subset* of $\{x_1, x_2, x_3, \ldots, x_n\}$ that contains *only* the scores of the child nodes deemed critical* to the parent node, then define the critical average of $x_1, x_2, x_3, \ldots, x_n$ as follows:

$$Crit\ Ave = \lambda\, Max\{A\} + (1-\lambda)\, Average\,\{x_1, x_2, x_3, \ldots, x_n\} \qquad (4.81)$$

where $0 \leq x_i \leq 100$ for all $i = 1, 2, 3, \ldots, n$, and λ is a weighting function, such as the weighting function in Figure 4.53.

Next, we'll apply the critical average to the nodes in Figure 4.56 as the rule to measure the risk to Functionality 3.22. Suppose the EWXT Program is deemed the *only* critical supplier to Functionality 3.22; thus, $A = \{RS_{3.221}\}$ in this case. From this, it follows that

$$\begin{aligned} Risk\ Score(Functionality\ Node_{3.22}) &= RS_{3.22} \\ &= \lambda Max\,\{A\} + (1-\lambda)\, Average\,\{RS_{3.221}, RS_{3.222}, RS_{3.223}\} \end{aligned} \qquad (4.82)$$

where λ is a weighting function. For convenience, use the weighting function in Figure 4.53. Then, from the risk scores in Figure 4.56 and Equation 4.82 we have

$$RS_{3.22} = (0.70)(84) + (1-0.70)\, Average\,\{84, 67.9, 26.08\} = 76.6$$

Thus, Functionality 3.22 has a high risk score, denoted by $RS_{3.22}$. A picture of this result is illustrated in Figure 4.57.

The high risk score for Functionality 3.22 is driven by the importance of the EWXT Program. According to the scale convention in Figure 4.52, Functionality 3.22 would also fall in the "RED" color band, as shown in Figure 4.57.

It is important that various analyses be conducted to examine the risk-drivers to Functionality 3.22. It can be shown that 88% of the high risk score of Functionality 3.22 is driven by the high risk score of the EWXT Program. The high risk score of the EWXT Program is driven by R1, R2, R4, R6, and R8. These five risk events collectively account for 93% of the EWXT Program's risk score.

*A child node's contribution to its parent node is *critical* if, without the contribution, the parent node's outcome objectives are not achieved or are unacceptably degraded.

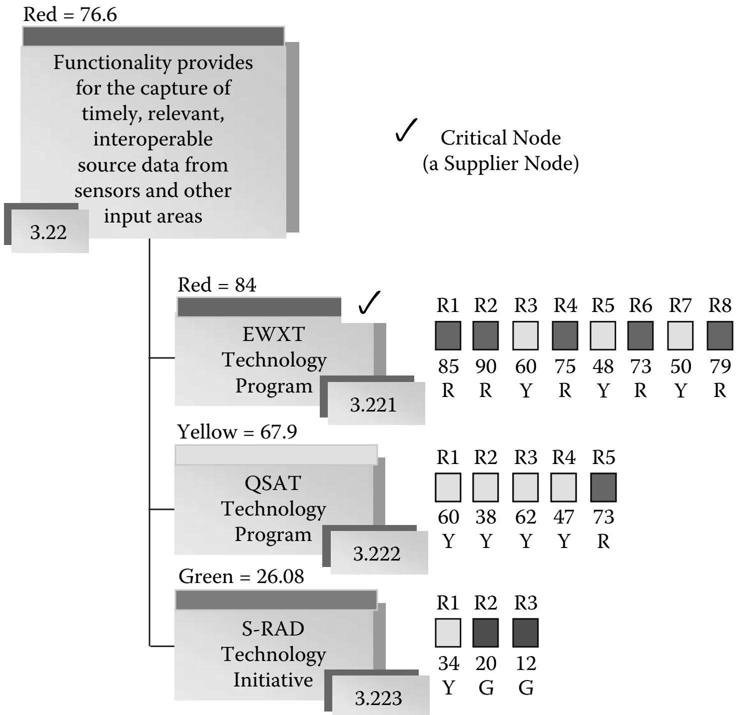


Figure 4.57: Risk measure derived for functionality 3.22.

These risk events signal where management attention is needed with respect to reducing the risk to Functionality 3.22. Not properly managing these risks, or not targeting them for management consideration, will further contribute to negative effects at higher dependency levels in the capability portfolio's hierarchy.

Measuring "Up": How Functionality Risks Affect Capability

The preceding presented ways to derive a measure of Functionality 3.22 risk as a function of its supplier risks. Shown in Figure 4.50, Functionality 3.22 is one of four functionality nodes to Capability 3.2. What about the other functionality nodes? How might their risk measures combine into an overall measure of risk to Capability 3.2? What ripple effects do these Tier 4 functionality risks have on dependent higher-level capability nodes in the capability portfolio's hierarchy? The following will address these and related questions.

Suppose risk measures for the other three functionality nodes to Capability 3.2 are shown along Tier 4 in Figure 4.58. These nodes are Functionality 3.21, 3.23,

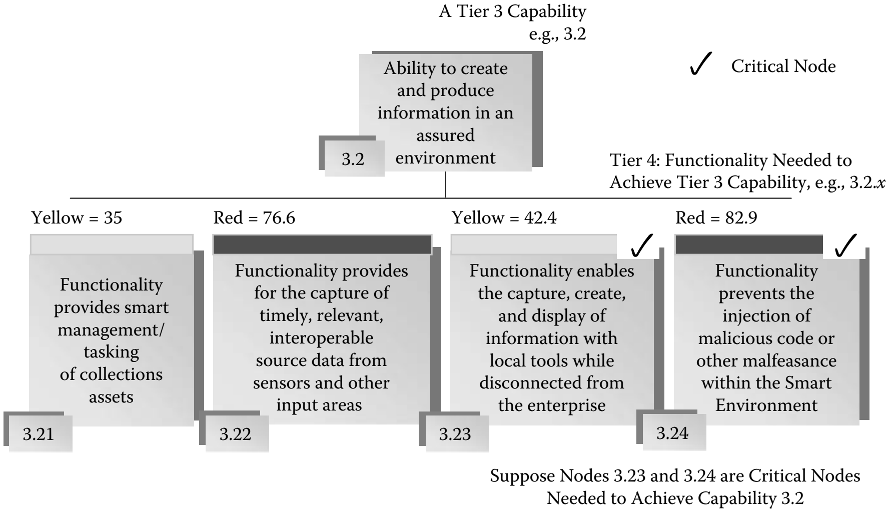


Figure 4.58: Risk measures for capability 3.2 functionality nodes.

and 3.24. Suppose their risk measures were also derived as a function of the risks their supplier nodes face, according to the same process just described. For convenience, we defer showing their supplier nodes to keep Figure 4.58 less visually complicated.

Next, we address a way to combine risk measures from all four functionality nodes in Figure 4.58 into an overall measure of risk to Capability 3.2? Here, we can again apply the critical average rule across the four Tier 4 functionality nodes to derive a measure of risk faced by Capability 3.2 — a Tier 3 node.

In Figure 4.58, *suppose* (in this case) Functionality 3.23 and 3.24 are deemed the critical functions to achieving Capability 3.2; thus, $A = \{RS_{3.23}, RS_{3.24}\}$ in this case. From this, it follows that

$$\begin{aligned} &Risk\ Score\ (Capability\ Node_{3.2}) = RS_{3.2} \\ &\quad = \lambda Max\{A\} + (1-\lambda)\,Average\{RS_{3.21}, RS_{3.22}, RS_{3.23}, RS_{3.24}\} \end{aligned} \tag{4.83}$$

where λ is a weighting function. For convenience, use the weighting function in Figure 4.53. Then, from the risk scores in Figure 4.58 and Equation 4.83 we have

$$RS_{3.2} = (0.70)(82.9) + (1-0.70)\,Average\{35, 76.6, 42.4, 82.9\} = 75.8$$

Thus, we conclude that Capability 3.2 has a high risk score. This is driven by the importance of Functionality 3.24. According to the scale convention in

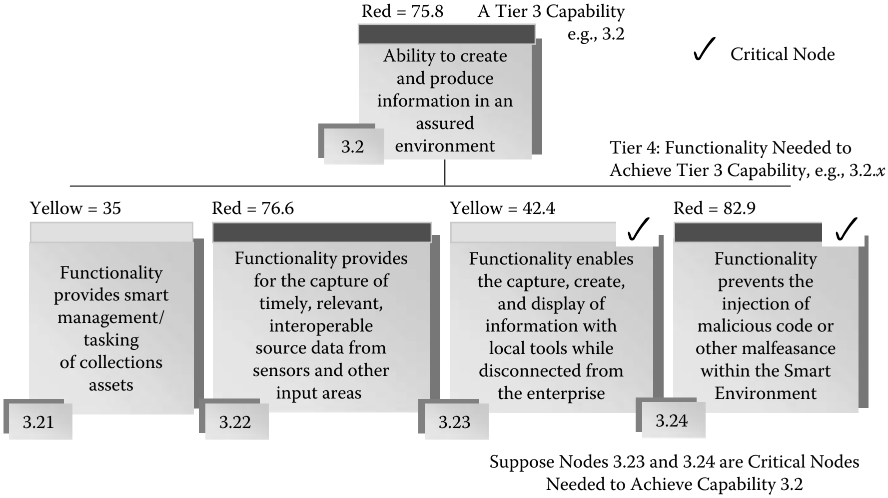


Figure 4.59: Risk measure for capability 3.2: Critical Average Rule.

Figure 4.52, Capability 3.2 would fall in the "RED" color band. The results of this discussion are illustrated in Figure 4.59.

Measuring "Up": How Capability Risks Affect the Capability Portfolio

The preceding discussion presents ways to derive a measure of Capability 3.2 risk as a function of its Functionality risks. Shown in Figure 4.60, Capability 3.2 is one of four capability nodes to Information Assurance, a Tier 2 capability area. What about the other capability nodes? How might their risk measures combine into an overall measure of risk to Tier 2 Information Assurance? What ripple effects do these Tier 3 capability risks have in the capability portfolio's hierarchy? The following will address these and related questions.

Suppose risk measures for the other three capability nodes to the Tier 2 node Information Assurance are shown in Figure 4.61. These nodes are Capability 3.1, 3.3, and 3.4. Suppose their risk measures were also derived as a function of the risks their functionality nodes face, according to the same process just described. For convenience, we defer showing their functionality nodes to keep Figure 4.61 less visually complicated.

We will again apply the critical average rule to combine risk measures from all four capability nodes, in Figure 4.61. This will produce an overall measure of risk to the Tier 2 node Information Assurance.

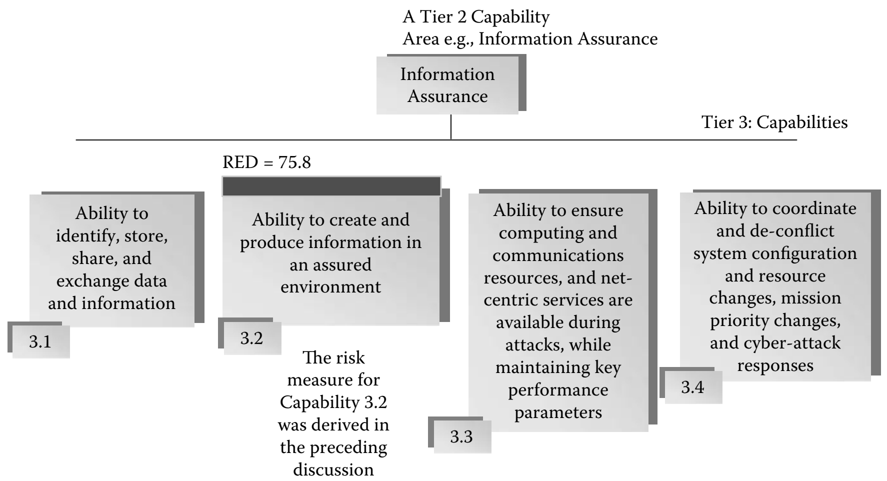


Figure 4.60: Information assurance: a Tier 2 capability area.

In Figure 4.61, *suppose* (in this case) Capabilities 3.1, 3.3, and 3.4 are deemed the critical capabilities to achieving Tier 2 Information Assurance. Given this, and applying the critical average rule, set A is equal to $A = \{RS_{3.1}, RS_{3.3}, RS_{3.4}\}$. From this, the max of set A is

$$Max\,\{A\} = Max\,\{RS_{3.1}, RS_{3.3}, RS_{3.4}\} = 38.1$$

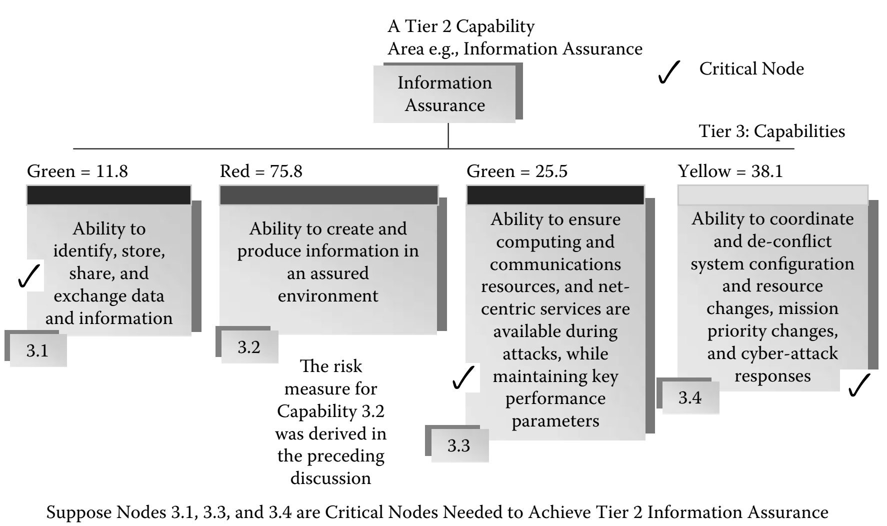


Figure 4.61: Information assurance: capability risk measures.

Thus, from Equation 4.81 we have

$$Risk\ Score\ (Information\ Assurance\ Node) = RS_{IA}$$
$$= \lambda Max\{A\} + (1-\lambda)\, Average\, \{RS_{3.1}, RS_{3.2}, RS_{3.3}, RS_{3.4}\} \quad (4.84)$$

where λ is a weighting function. For convenience, use the weighting function in Figure 4.53. Then, from the risk scores in Figure 4.61 and Equation 4.84 we have

$$RS_{IA} = (0.381)(38.1) + (1-0.381)\, Average\, \{11.8, 75.8, 25.5, 38.1\} = 38$$

We conclude the Tier 2 Information Assurance capability area has a moderate risk score. This is driven by the importance of Capability 3.4. According to the scale convention in Figure 4.52, the Tier 2 Information Assurance capability area would fall in the "YELLOW" color band. The results of this discussion are illustrated in Figure 4.62.

Suppose risk measures for the other three Tier 2 capability areas are shown in Figure 4.63. These nodes are Network Management, Enterprise Services, and Communications and Applications. Suppose their risk measures were derived as a function of the risks their Tier 3 capability nodes face, according to the same process just described. We can apply the critical average rule to combine risk measures from all four Tier 2 capability areas into an overall measure of

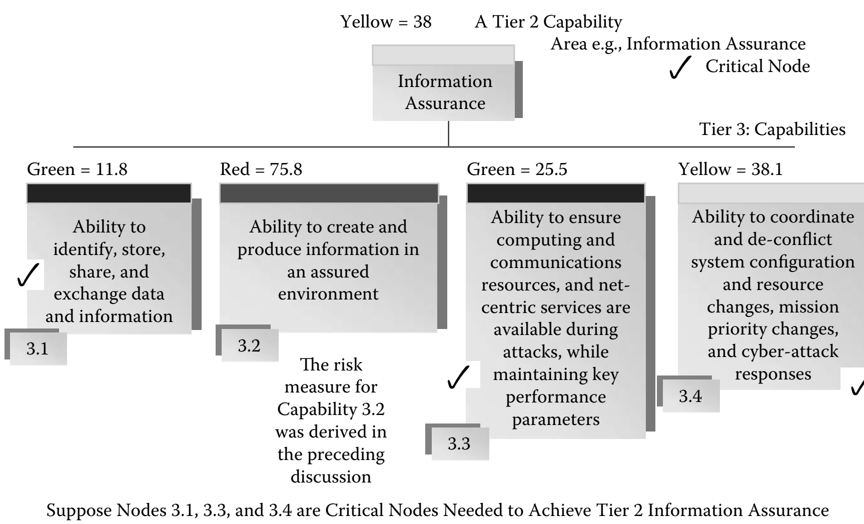


Figure 4.62: Information assurance: capability risk measures.

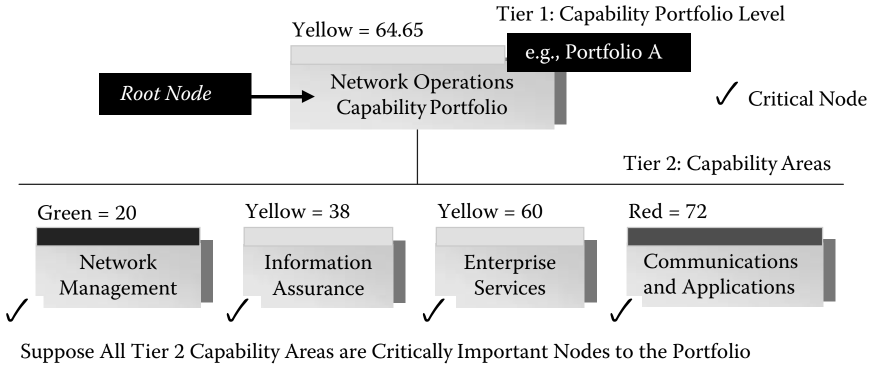


Figure 4.63: Network operations capability portfolio-level risk measure.

risk to the capability portfolio itself. Assume all four Tier 2 capability areas are critical to the capability portfolio. Figure 4.63 presents the results of this computation.

Hence, we see the Network Operations Capability portfolio is facing an overall moderate-level of risk with a risk measure of 64.65. According to the scale convention in Figure 4.52, the capability portfolio's overall risk measure places it in the "YELLOW" color band.

The preceding discussion presented an "algebra" designed to measure risk at any node in a capability portfolio when risk events originate from a capability portfolio's supplier-levels. Computational rules were defined and illustrated to show how risk measures derive, in part, from a series of rollup calculations. Risk measures derived from leaf nodes were rolled up to measure the risks of parent nodes. Risk measures derived for parent nodes were rolled up to measure the risk of the capability portfolio itself.

In the context of this formalism, the *number* of risk events associated with a supplier node does not fully drive the magnitude of its risk measure. Consider the max average rule. This rule is purposefully designed to weight more heavily risk events, in a set of events, with higher risk measures (risk scores) than those in the set with lower risk measures. Although risk scores of all risk events associated with a supplier node are included in the max average, *their effect on the node's overall risk measure is controlled by the shape or form of the weighting function* λ. Because of this, each risk event does not necessarily contribute equally to the

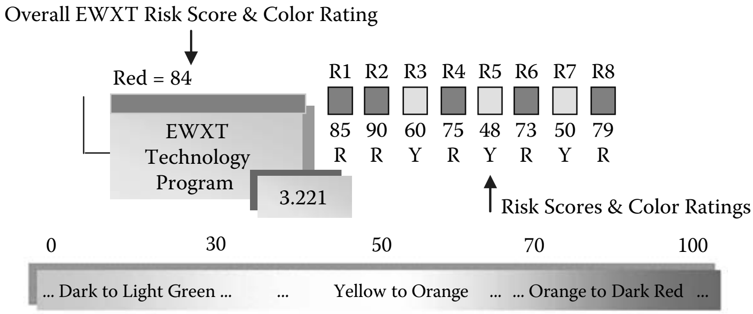


Figure 4.64: Overall EWXT program risk score (max ave, R = Red, Y = Yellow).

supplier node's overall risk measure. A supplier node with a set of five risk events can have a higher risk measure than one with a set containing more than five risk events *and vice versa*.

Thus, with the max average rule it is important to design the shape or form of its weighting function to capture the team's (or decision-maker's) preferences for the degree the maximum score should influence the overall score. Two weighting functions are shown in Figures 4.19 and 4.53. Many other shapes are possible.

The *max average* rule applied in the context of Figure 4.54 operates, under certain conditions, as a decision-maker's "alert function." Consider Figure 4.54 (shown as Figure 4.64 for convenience). Here, the supplier node's risk measure was 84 given the eight risks R1 through R8. Suppose management actions were taken such that R3 through R8 were eliminated from this supplier node's risk set. With this, the EWXT Technology Program would now have a risk measure of 89.25.

Why did this supplier node's risk measure increase despite the elimination of all but two of its risks? The answer includes the following: (1) management actions eliminated R3 through R8 — but they did not eliminate the two most serious risks, R1 and R2, from the node's risk set; and (2) the max average rule operates only on the risk set presented; so, even though R3 through R8 were eliminated, the max average rule only "sees" a supplier node with two serious risks R1 and R2.

The fact that the risk measure increased is noteworthy, but not as important as the result that the node remained in the "Red" risk color band in this example. Thus, the max average rule can be *tuned* to alert management when a supplier node *still* faces a high degree of risk because of the presence of even just a few very serious risks — despite the elimination of less serious ones from the set.

What about risks to capabilities when risk events originate from non-supplier-related sources or conditions? How can these risks be considered in a capability portfolio risk assessment?

Risks that threaten capabilities to be delivered by a capability portfolio can originate from sources other than those that affect only the portfolio's suppliers. These events can directly attack one or more capability nodes in a capability portfolio's hierarchy. For example, uncertainties in geo-political landscapes may impact operational demands on capabilities that stress planned performance.

Dependencies between capability portfolios in families of portfolios, such as those that constitute an enterprise, are also potential risk sources. Here, outcome objectives for capabilities delivered by one capability portfolio may depend on the performance of capabilities delivered by another capability portfolio. Identifying risk events from non-supplier-related sources and capturing their contribution to a capability node's risk measure is an important consideration in a capability portfolio's risk assessment and analysis process.

This process, as described, provides ways to separate, track, and report risks faced by capability nodes, as a function of the many sources of risk affecting these nodes and ultimately the capability portfolio. In practice, it is recommended that supplier and non-supplier measures of capability risk be separately derived, tracked, and reported to the capability portfolio's management team. In addition, each risk should be tagged according to its type (see Table 4.27) and tracked in the capability portfolio's overall risk "population."

If the above is done, then a variety of management indicators can be developed. These include (1) the frequency with which specific types of risk affect capability nodes and (2) the degree a capability node's risk measure is driven by supplier versus non-supplier source conditions, including understanding the nature and drivers of these conditions.

We end this discussion with a summary of the information needed to implement capability portfolio risk management. The chapter concludes with a perspective on capability portfolio risk management and its relationship to the management of risk in engineering the enterprise.

Capability Portfolio Risk Management: Information Needs and Considerations

Risk management in a capability portfolio context has unique and thought challenging information needs. These needs can group into two categories. The first category *addresses capability value*. The second category *addresses supplier contributions, criticality, and risks* as they relate to enabling the portfolio to deliver capability.

Information needs that *address capability value* include the following:

- For each Tier 3 capability, shown in Figure 4.48, what standard (or outcome objective) must each capability meet by its scheduled delivery date?
- For each Tier 3 capability, what is the source basis for its standard (or outcome objective)? Does it originate from user-driven needs, policy-driven needs, model-derived values, a combination of these, or from other sources?
- For each Tier 3 capability, what extent does the standard (or outcome objective) for one capability depend on others meeting their standards (or outcome objectives)?

Information needs that *address supplier contributions, criticality, and risks* include the following:

- For each Tier 3 capability, which Technology Programs and Technology Initiatives are contributing to that capability?
- For each Tier 3 capability, what (specifically) are the contributions of its suppliers?
- For each Tier 3 capability, how do supplier contributions enable the capability to achieve its standard (or outcome objective)?
- For each Tier 3 capability, which Technology Programs and Technology Initiatives are critical contributors in enabling the capability to achieve its standard (or outcome objective)?
- Given the above, what risks originate from (or are associated with) suppliers that, if these events occur, negatively affect their contributions to capability?

A similar set of information needs can be crafted for risk events that originate from non-supplier-related sources or conditions.

Measuring, tagging, and tracking risk events in the ways described aids management with identifying courses of action, specifically, whether options exist to attack risks directly at their sources or to engage them by deliberate intervention actions — actions aimed at lessening or eliminating their potential capability consequences.

Process tailoring, socialization, and establishing governance protocols are critical considerations in engineering risk management. Ensuring these aspects succeed is time well spent. With this, effective and value-added engineering management practices can be institutionalized — practices that enable capability portfolio outcomes, and ultimately those of the enterprise, to be achieved via risk-informed resource and investment management decisions.

To conclude, the approach presented for capability portfolio risk management provides a number of beneficial and actionable insights. These include the following:

- Identification of risk events that threaten the delivery of capabilities needed to advance goals and capability outcome objectives.
- A measure of risk for each capability derived as a function of each risk event's occurrence probability and its consequence.
- An analytical framework and logical model within which to structure capability portfolio risk assessments — one where assessments can be combined to measure and trace their integrative effects on engineering the enterprise.
- Through the framework, ways to model and measure risk as capabilities are time-phased across incremental capability development approaches.
- Decision-makers provided the trace basis and the event drivers behind all risk measures derived for any node at any level of the capability portfolio's hierarchy. With this, capability portfolio management has visibility and supporting rationales for identifying where resources are best allocated to reduce (or eliminate) risk events that threaten achieving goals and capability outcome objectives.

4.6.3 The "Cutting Edge"

The preceding section presented an analytical framework and computational model for assessing and measuring risk in the engineering of enterprise systems. It illustrated one among many potential ways to represent, model, and measure risk when engineering an enterprise from a capability portfolio perspective.

Few protocols presently exist for measuring capability risk in the context of capability portfolios. Additional research is needed on such protocols and how to customize them to specific supplier–provider relationships. Here, concepts from graph theory might be used to visualize and model a capability portfolio's supplier–provider topology. Computational algebras might then be designed to generate measures of capability risk unique to that portfolio's topology.

Protocols are also needed to capture and measure horizontal and vertical dependencies among capabilities and suppliers within capability portfolios and across families of capability portfolios that make an enterprise. With this, the ripple effects of failure in one capability (or supplier) on other dependent capabilities (or suppliers) or portfolios could be formally measured. Developing ways to capture and measure these effects would enable designs to be engineered that minimize dependency risks. This might lessen or even avoid potentially cascading negative effects that dependencies can have on the timely delivery of enterprise services to consumers.

Additional areas at the "cutting edge" include the following:

- How time-phasing capability delivery to consumers should be factored into risk assessment, measurement, and management formalisms.
- How to approach risk measurement and management in enterprises that consists of dozens of capability portfolios with hundreds of supplier programs. Here, the idea of representing large-scale enterprises by *domain capability portfolio clusters* might be explored and a new concept of *portfolio cluster risk management* might be developed.
- How to design decision analytic methodologies to measure risk criticality that captures each risk's multi-consequential impacts and dependencies across enterprise-wide capabilities.

The materials in section 4.6 aimed to bring conceptual understandings of the enterprise engineering problem space into view. With this, risk management theory and practice for engineering enterprises can evolve.

This topic falls at the interface between risk management methods for engineering traditional systems with those needed for engineering enterprise systems. Recognizing this interface and then addressing its challenges is an essential step toward discovering new methods and new practices uniquely designed to successfully manage risk in engineering an enterprise.

N. W. Dougherty, past president of the American Society for Engineering Education (1954–1955), once said: "*the ideal engineer is a composite . . . he is not a scientist, he is not a mathematician, he is not a sociologist or a writer; but he may use the knowledge and techniques of any or all of these disciplines in solving engineering problems*." That was true then and is even truer today.

Questions and Exercises

1. Areas of risk common to an engineering system project are described in Table 4.1. For each area, describe strategies that might be applied to lessen or eliminate these risks. Are some strategies "better" than others? What factors should be considered in deciding "better"?

2. Review the guidelines for identifying risks presented in Table 4.2. What might be added or emphasized with respect to these guidelines?

3. What part of a risk statement's *Condition-If-Then* formalism (see Figure 4.2 or Figure 4.14) identifies a clear intervention target for directly attacking the risk?

4. Study the ordinal risk matrix approach in section 4.3.1 and design a strictly *probability* averse 5×5 matrix. Compare and contrast this matrix with the matrix developed and presented in Figure 4.8.

5. (A) Given the information in the following table, rank each risk event from highest-to-lowest consequence code. (B) Plot each risk event by its probability versus its consequence code. (C) How might probability and consequence code be used *together* to identify three highest priority risk events?

Risk				Consequence Areas & Assessments		
Event	Probability Assessment				Technical	
ID #	Cardinal	Ordinal	Cost	Schedule	Performance	Programmatics
1	0.95	5	4	5	5	4
2	0.50	3	2	2	3	5
3	0.25	2	5	5	5	4
4	0.65	3	3	3	1	4
5	0.35	2	2	4	4	5
6	0.95	5	4	4	1	4
7	0.75	4	3	3	5	2
8	0.90	5	5	5	1	4
9	0.45	3	3	5	5	3
10	0.75	4	5	3	5	3

Table for Problem 5.

Assume the values in the table for problem five are defined in accordance with the scales provided in Tables 4.3 through 4.7, respectively.

Extra credit: Program a computer application to operate as a general model of the consequence code ranking approach.

6. (A) Apply the Borda algorithm to the risk events in Problem 5 to derive an ordinal risk ranking as a function of each risk event's occurrence probability and consequence. (B) Compare and contrast the Borda risk ranking with the ranking produced by applying the ordinal risk matrix in Figure 4.10C. (C) Discuss advantages and disadvantages of the Borda ranking approach with those associated with an ordinal risk matrix approach.

   Extra credit: Program a computer application to operate as a general model of the Borda risk ranking approach.

7. Apply the value functions in Figure 4.12, Figure 4.13, and Figure 4.17 to the risk event data in Problem 5 to answer the following:
   (A) Use Formulation A (Equation 4.11) to compute each risk event's *Risk Score*.
   (B) Use Formulation C (Equation 4.13) to compute each risk event's *Risk Score*.
   (C) Generate a scatter plot showing each risk event by its occurrence probability and its overall consequence score, V_{Impact}, where V_{Impact} is defined according to Formulation A and Formulation C.

For this problem, assume importance weight assessments for each consequence area is as follows: technical performance w_3 is twice as important as cost w_1; cost w_1 is twice as important as schedule w_2; cost w_1 is twice as important as programmatics w_4.

8. (A) Review Case Discussion 4.1a and work through the computations to the extent practical with available computing software. (B) Discuss the importance of probabilistic independence. (C) In Case Discussion 4.1a, how might the expected utility approach be modified if probabilistic independence cannot be assumed.

9. Discuss how using expected utilities, described in section 4.3.4, compares to an approach based on the sensitivity of results to deterministic changes in values of key input variables (i.e., instead of probability distributions to represent a range of possible values for each criterion, as in Table 4.14).

10. Case Discussion 4.2 illustrated an approach for monitoring the progress of risk management plans. Assessment dates were shown in Tables 4.15 through 4.18 that reflect the progress of each activity for a hypothetical risk management plan. From the PIPD and *Probability of Success* assessments shown in these tables derive the corresponding API and AAPI values, respectively.

    Extra credit: Program a computer application to operate as a general model of the risk management performance index approach described in section 4.4.2.

11. Review the approach in section 4.5.1 for computing a technical performance risk index measure. Derive the $\text{TRI}_{ti,\text{All}}$ measure for the six time periods in Table 4.23.

    Extra credit: Program a computer application to operate as a general model that produces the technical performance risk index measure described in section 4.5.1.

12. In the following figure, show the *critical* or *max average* rules generate a risk measure of 64.65 for the node labeled *Network Operations Capability Portfolio*.

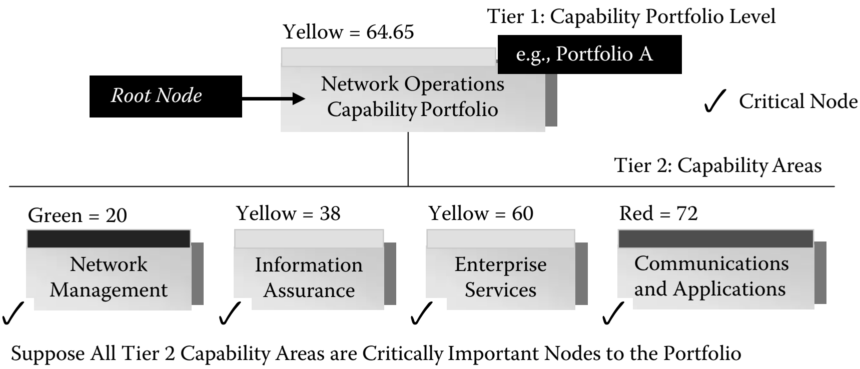


Figure for Problem 12.

13. Suppose the following figure represents a portion of a capability portfolio defined as part of engineering an enterprise system. Given the information shown, apply the same computational formalisms presented in section 4.6.2 to derive a risk measure for Capability 3.2. What risks are driving this measure?

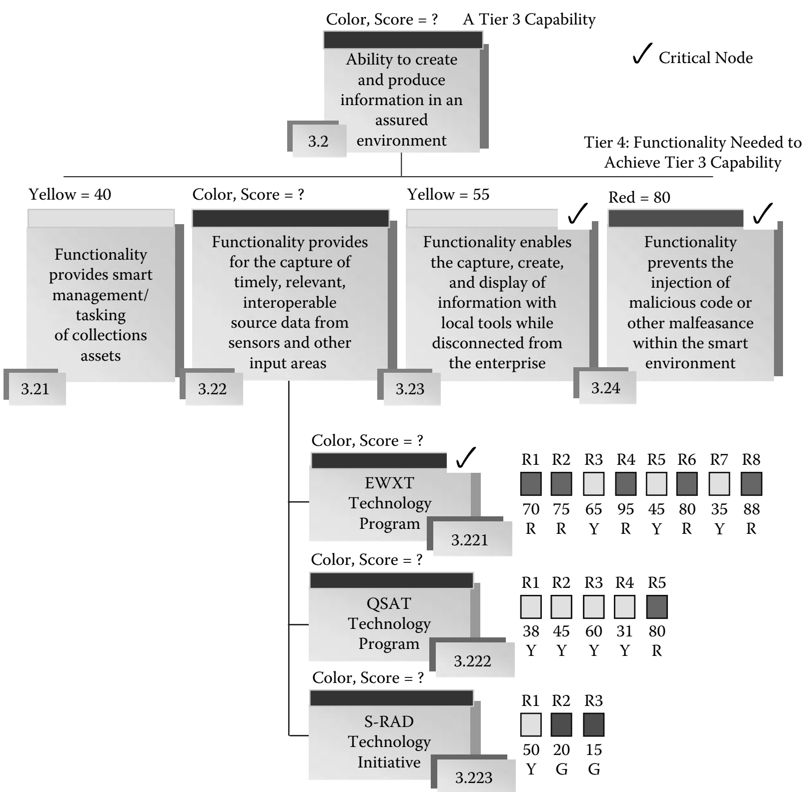


Figure for Problem 13.

Questions 14–20: For Research Projects or Class Discussions

14. **Identifying Which Risks to Mitigate:** Section 4.4.3 introduced a technique to identify which combination of risks to mitigate that maximize reducing a project's risk exposure while not exceeding an overall mitigation budget. The technique involved solving the classic knapsack problem. **Think about** how to modify the knapsack problem when the mitigation of one risk requires the mitigation of one (or more) other risks. **Think about** how to include the time when a risk might occur in the selection of which risks to mitigate.

Questions Related to Risk Management in Engineering Enterprise Systems

15. **Risk Measurement:** Section 4.6.2 presented a framework and algebra for assessing and measuring capability risk in the context of capability portfolios, defined for engineering an enterprise. Specifically, linear additive protocols were presented as one way to measure capability risk due to supplier-provider relationships in a portfolio. **Think about** other possible risk measurement protocols, or variations on an additive approach, that might be designed for this problem space. Demonstrate advantages and disadvantages of various measurement protocols and conditions when one is preferred over another.

16. **Capturing Dependencies:** Related to the above, protocols are needed to capture and measure horizontal and vertical dependencies among capabilities and suppliers within capability portfolios and across families of capability portfolios that make up an enterprise. With this, the ripple effects of failure in one capability (or supplier) on other dependent capabilities (or suppliers) or portfolios could be formally measured. **Think about** ways to capture, model, and measure these effects so designs can be engineered that minimize dependency risks and their potentially cascading negative effects on the timely delivery of enterprise services to consumers.

17. **Time-Phase Considerations: Think about** how the time-phasing of capability delivery to consumers of enterprise services might be planned and then factored into risk assessment, measurement, and management formalisms.

18. **Enterprise Scale Considerations: Think about** how to approach risk measurement and management in enterprises that consist of dozens of capability

portfolios with hundreds of supplier programs. Here, a new idea of representing large-scale enterprises by *domain capability portfolio clusters* might be explored and a new concept of *portfolio cluster risk management* might be developed.

19. **Risk-Adjusted Benefit Measure:** Consider a capability portfolio being managed through a supplier-provider approach, as discussed in section 4.6.2. Suppose a capability portfolio manager must select investing in suppliers that offer the most benefit to achieving capability, in terms of desired outcomes. **Think about** how to measure investment benefit but how to adjust this measure to account for risks each supplier may face in delivering their contribution to the capability portfolio's desired capability outcomes.

20. **Governance: Think about** ways to structure management and oversight boards for an enterprise risk management process. Define necessary process participants, the decision authority chain and its operations, and the roles of stakeholders in the process as consumers of enterprise services.

References

[1] Bahnmaier, W. W., Ed., June 2003. *Risk Management Guide for DOD Acquisition*, 5th ed., Version 2.0, Department of Defense: Defense Acquisition University Press, Fort Belvoir, VA.

[2] Garvey, P. R., April 2001. "Implementing a Risk Management Process for a Large Scale Information System Upgrade — A Case Study," *INSIGHT*, Vol. 4, Issue 1, International Council on Systems Engineering (INCOSE).

[3] Garvey, P. R., January 2005. "System-of-Systems Risk Management: Perspectives on Emerging Process and Practice," The MITRE Corporation, MP 04B0000054.

[4] Saari, D. G., 2001. *Decisions and Elections: Explaining the Unexpected*, Cambridge University Press, New York.

[5] Garvey, P. R., January 2000. *Probability Methods for Cost Uncertainty Analysis: A Systems Engineering Perspective*, Marcel Dekker, New York.

[6] Garvey, P. R., Cho, C. C., Spring 2003. "An Index to Measure a System's Performance Risk," *The Acquisition Review Quarterly* (ARQ), Vol. 10, No. 2.

[7] Blanchard, B. S., and W. J. Fabrycky. 1990. *Systems Engineering and Analysis*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, Inc.

[8] Garvey, P. R., Cho, C. C., Winter 2005. "An Index to Measure and Monitor a System-of-Systems' Performance Risk," *The Acquisition Review Journal* (ARJ).

[9] Browning, T. R., Deyst, J. J., Eppinger, S. D., November 2002. "Adding Value in Product Development by Creating Information and Reducing Risk," *IEEE Transactions on Engineering Management*, Vol. 49, No. 4.

[10] Allen, T., Nightingale, D., Murman, E., March 2004. "Engineering Systems: An Enterprise Perspective," An Engineering Systems Monograph, Engineering Systems Division, The Massachusetts Institute of Technology.

[11] Rebovich, G., Jr., November 2005. "Enterprise Systems Engineering Theory and Practice, Volume 2, Systems Thinking for the Enterprise: New and Emerging Perspectives," The MITRE Corporation. http://www.mitre.org/work/tech_papers/tech_papers_06/05_1483/05_1483.pdf.

[12] Gharajedaghi, J., 1999. *Systems Thinking: Managing Chaos and Complexity — A Platform for Designing Business Architecture*, Woburn, MA: Butterworth-Heinemann.

[13] White, B. E., October 2006. "Fostering Intra-Organizational Communication of Enterprise Systems Engineering Practices," The MITRE Corporation, National Defense Industrial Association (NDIA), 9th Annual Systems Engineering Conference, October 23–26, 2006, Hyatt Regency Islandia, San Diego.

[14] Office of the Secretary of Defense, October 2005: *Net-Centric Operational Environment Joint Integrating Concept*, Version 1.0, Joint Chiefs of Staff, 31 October 2005, Joint Staff, Washington, D.C. 20318-6000; reference http://www.dod.mil/cio-nii/docs/netcentric_jic.pdf.

[15] Government Accountability Office (GAO), July 2004. "Defense Acquisitions: The Global Information Grid and Challenges Facing its Implementation," GAO-04-858.

Appendix A

A Geometric Approach for Ranking Risks

A.1 Introduction

This appendix introduces a geometric approach that can be used to rank risk events on the basis of their "performance" across multiple evaluation criteria. This approach has a number of uniquely desirable features and it can be used in conjunction with value function approaches described in Chapter 3 and Chapter 4.

Technique for Order Preference by Similarity to Ideal Solution

The geometric approach introduced in this appendix is known as TOPSIS, which stands for *Technique for Order Preference by Similarity to Ideal Solution*. TOPSIS is known in the decision sciences literature as an "ideal point" multiple criteria decision analysis method. It generates indices that order a set of competing alternatives from most-to least-preferred (or desirable) as a function of multiple criteria. TOPSIS was developed in 1981 by Hwang and Yoon [1].

The ideal point represents a hypothetical alternative that consists of the most desirable weighted normalized levels of each criterion across the set of competing alternatives. The alternative closest to the ideal point performs best in the set. Separation from the ideal point is measured geometrically by a Euclidean distance metric. This is illustrated in Figure A.1.

Figure A.1 shows two alternatives A_1 and A_2 in relation to two benefit criteria or attributes (Attribute 1 and Attribute 2). Here, A_1 is closest to the ideal solution A^* but A_2 is farthest from the negative ideal solution A^-. So, which one do you choose?

TOPSIS is an ideal point method that ensures the chosen alternative is simultaneously closest to the ideal solution *and* farthest from the negative ideal solution. TOPSIS chooses the alternative whose performance across all criteria maximally matches those that compose the ideal solution.

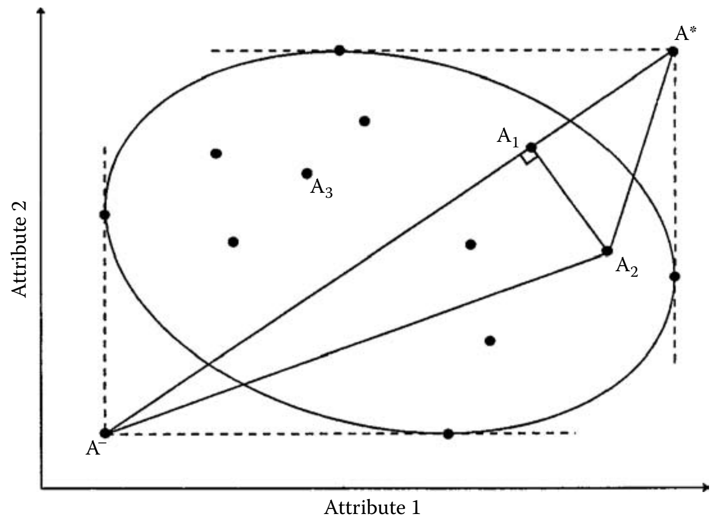


Figure A.1: Euclidean distances to positive and negative ideal solutions. (Reprinted by permission of Sage Publications, Inc. [1])

TOPSIS assumes each attribute (or criterion) can be characterized by either monotonically increasing or decreasing utility. Here, we seek to maximize attributes that offer a benefit and minimize those that incur a cost. TOPSIS generates an index that rank-orders competing alternatives from most- to least-desired on the relative distance of each to the ideal.

Objective Weighting: The Entropy Method

Table A.1 presents a generalized decision or performance matrix of alternatives. Here, the performance of an alternative is evaluated across competing criteria. The attractiveness of an alternative to a decision-maker is a function of the performance of each alternative across these criteria.

*Entropy** is a concept found in information theory that measures the uncertainty associated with the expected information content of a message. It is also used in decision science to measure the amount of decision information contained and transmitted by a criterion.

*In information theory, entropy measures the uncertainty associated with the expected information content of a message [ref: Shannon, Claude, E., 1948. *A Mathematical Theory of Communication*]; http://en.wikipedia.org/wiki/Claude_Elwood_Shannon, Bell System Technical Journal, 1948.

TABLE A.1: A Traditional Decision or Performance Matrix of Alternatives

	Criteria & Weights				
Decision	C_1	C_2	C_3	$\cdots$	C_n
Alternative	w_1	w_2	w_3	$\cdots$	w_n
A_1	x_{11}	x_{12}	x_{13}	$\cdots$	x_{1n}
A_2	x_{21}	x_{22}	x_{23}	$\cdots$	x_{2n}
A_3	x_{31}	x_{32}	x_{33}	$\cdots$	x_{3n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_m	x_{m1}	x_{m2}	x_{m3}	$\cdots$	x_{mn}
	A Decision Matrix				

The amount of decision information contained and transmitted by a criterion is driven by the extent the performance (i.e., "score") of each alternative is distinct and differentiated by that criterion. When alternatives (in a decision matrix) all have the *same* performance for a criterion, we say the criterion is unimportant. It can be dropped from the analysis because it is not transmitting distinct and differentiating information. The more distinct and differentiated the performance of competing alternatives on a criterion, the greater the amount of decision information contained and transmitted by that criterion; hence, the greater its importance weight.

In decision science, entropy is used to derive *objective* measures of the relative importance of each criterion (i.e., its weight) as it influences the performances of competing alternatives. If desired, prior subjective weights can be folded into objectively-derived entropy weights. This is discussed later.

Equations for the TOPSIS Method

Applying TOPSIS consists of the following steps and equations [1].

Step 1. Normalize the values in the decision matrix of alternatives (Table A.1). One way to do this is to compute r_{ij} where

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad i = 1, \ldots, m; \quad j = 1, \ldots, n$$

Step 2. From step 1, compute weighted normalized values. This can be done by computing v_{ij}, where

$$v_{ij} = w_j r_{ij} \quad i = 1, \ldots, m; \quad j = 1, \ldots, n$$

and w_j is the weight of the jth attribute (criterion). In this step, w_j could be replaced by the *entropy weight*, which is discussed later in this appendix.

Step 3. Derive the positive A^* and the negative A^- ideal solutions, where

$$\begin{aligned} A^* &= \{v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^*\} \\ &= \{(\max_i v_{ij} | j \in J_1), (\min_i v_{ij} | j \in J_2) | i = 1, \ldots, m\} \\ A^- &= \{v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^-\} \\ &= \{(\min_i v_{ij} | j \in J_1), (\max_i v_{ij} | j \in J_2) | i = 1, \ldots, m\} \end{aligned}$$

where J_1 is the set of benefit attributes and J_2 is the set of cost attributes.

Step 4. Calculate separation measures between alternatives, as defined by the n-dimensional Euclidean distance metric.

The separation from the positive-ideal solution A^* is given by

$$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2} \quad i = 1, \ldots, m$$

The separation from the negative-ideal solution A^- is given by

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2} \quad i = 1, \ldots, m$$

Step 5. Calculate similarities to positive-ideal solution, as follows:

$$0 \leq C_i^* = \frac{S_i^-}{(S_i^* + S_i^-)} \leq 1 \quad i = 1, \ldots, m$$

Step 6. Choose the alternative in the decision matrix with the maximum C_i^* and rank these alternatives from most- to least-preferred according to C_i^* in descending order. The closer C_i^* is to one the closer it is to the positive-ideal solution. The further C_i^* is from one the further it is from the positive-ideal solution.

Equations for the Entropy Weighting Method

The following steps present the equations for computing entropy-derived objective weights used to derive the "most-preferred" alternative in a decision matrix.

Step 1. From the decision matrix in Table A.1, compute p_{ij} where

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad i = 1, \ldots m; \quad j = 1, \ldots n$$

Step 2. Compute the entropy of attribute (criterion) j as follows:

$$0 \leq E_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} p_{ij} \ln p_{ij} \leq 1 \quad i = 1, \ldots m; \quad j = 1, \ldots n$$

Step 3. Compute the degree of diversification d_j of the information transmitted by attribute (criterion) j according to $d_j = 1 - E_j$

Step 4. Compute the entropy-derived weight w_j as follows:

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \quad j = 1, \ldots, n$$

If the decision-maker has prior subjective importance weights λ_j for each attribute (criterion), then this can be adapted into w_j as follows:

$$w_j^{\bullet} = \frac{\lambda_j w_j}{\sum_{j=1}^{n} \lambda_j w_j} \quad j = 1, \ldots, n$$

Application of TOPSIS Ranking Risks

Here, we describe how TOPSIS and the entropy weighting method can be applied to derive a most- to least-critical risk ranking from a set of identified risk events.

TABLE A.2: A "Risk Event" Decision Matrix

	Probability & Consequence Criteria				
Set of	**Prob**	C_1	C_2	$\cdots$	C_n
Risk Events	w_1	w_2	w_3	$\cdots$	w_n
Risk Event_1	x_{11}	x_{12}	x_{13}	$\cdots$	x_{1n}
Risk Event_2	x_{21}	x_{22}	x_{23}	$\cdots$	x_{2n}
Risk Event_3	x_{31}	x_{32}	x_{33}	$\cdots$	x_{3n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
Risk Event_m	x_{m1}	x_{m2}	x_{m3}	$\cdots$	x_{mn}
	A "Risk Event" Decision Matrix				

To begin, we first write the generalized decision matrix in Table A.1 into the form given in Table A.2.

In Table A.2, we have a set of risk events instead of alternatives "competing" for the position of "most-critical" to a project. The "most-critical" risk event has the highest occurrence probability *and* the highest consequences (or impacts) to the project. Thus, the higher these indicators across the matrix in Table A.2 the more critical is the associated risk event to the project.

Consider the following. Suppose we have seven risk events given in Table A.3. Suppose the performance of these risks, in terms of their occurrence probabilities and consequences to a project, are given in the columns of Table A.3. Suppose the consequence criteria values (in Table A.3) derive from the value functions in Figure 3.31 (Chapter 3). From these data, the TOPSIS equations will be applied to derive a score (or index) to rank-order each risk event from most- to least-critical to the project.

Applying each step of TOPSIS and the entropy weighting method described above produces the scores in the right-most column in Table A.4. Deriving these scores is straightforward and left as an exercise for the reader.

From Table A.4, the most- to least-critical risk ranking can be seen as follows:

Risk Event 4 (Least-Critical) $<$ Risk Event 3 $<$ Risk Event 5 $<$
Risk Event 6 $<$ Risk Event 7 $<$ Risk Event 2 $<$ Risk Event 1 (Most-Critical)

Entropy weighting of these data revealed the criterion *Technical Performance Impact* transmitted the most distinct and differentiating information from all

TABLE A.3: An Illustrative "Risk Event" Decision Matrix

	Consequence Criteria				
	Occurrence Probability	**Cost Impact**	**Schedule Impact**	**Technical Performance Impact**	**Programmatic Impact**
Risk Event 1	0.75	0.770	0.880	0.600	0.789
Risk Event 2	0.95	0.920	0.750	0.333	0.474
Risk Event 3	0.55	0.500	0.500	0.133	0.211
Risk Event 4	0.25	0.500	0.630	0.133	0.474
Risk Event 5	0.45	0.250	0.250	0.133	0.789
Risk Event 6	0.15	0.150	0.750	0.600	0.474
Risk Event 7	0.90	0.350	0.750	0.333	0.211

TABLE A.4: TOPSIS Scores for the Risk Event Data in Table A.3

	Consequence Criteria					
	Occurrence Probability	**Cost Impact**	**Schedule Impact**	**Technical Performance Impact**	**Programmatic Impact**	**TOPSIS Score**
Risk Event 1	0.75	0.770	0.880	0.600	0.789	0.849
Risk Event 2	0.95	0.920	0.750	0.333	0.474	0.668
Risk Event 3	0.55	0.500	0.500	0.133	0.211	0.305
Risk Event 4	0.25	0.500	0.630	0.133	0.474	0.267
Risk Event 5	0.45	0.250	0.250	0.133	0.789	0.306
Risk Event 6	0.15	0.150	0.750	0.600	0.474	0.463
Risk Event 7	0.90	0.350	0.750	0.333	0.211	0.474

others (in this case). Thus, this criterion has the most weight or influence on the derivation of the risk event rank-order positions shown above.

References

[1] Hwang, Ching-Lai, Yoon, K. Paul, 1995. *Multiple Attribute Decision Making: An Introduction*, Sage University Paper Series in Quantitative Applications in the Social Sciences, 07-104, Thousand Oaks, CA.

Appendix B

Success Factors in Engineering Risk Management

. . . If you don't actively attack the risks, they will actively attack you.

T. Gilb, *Principles of Software Engineering Management*, 1988

The following lists minimum conditions needed to initiate and continuously execute risk management as a meaningful, value-added, engineering management practice. With these, management increases the chance of identifying risks early so an engineering system's outcome goals and objectives may be achieved.

Requirements for Getting Risk Management Started

Senior leadership commitment and participation is required.

Stakeholder commitment and participation is required.

Risk management made a program-wide priority and "enforced" as such throughout the program's life cycle.

Technical and program management disciplines represented and engaged.

Risk management integrated and operationalized into the program's business and systems engineering plans and processes.

Twenty-One "Musts"

Risk management *must* be a priority for leadership and throughout the program's management levels.

Risk management *must* never be delegated to staff members or to organizations that lack authority or direct access to it.

A formal and repeatable risk management process *must* be present — one that is balanced in complexity and data needs, such that meaningful and actionable insights are produced with minimum burden.

The management culture *must* encourage and reward identifying risk by staff at all levels of program contribution.

Program leadership *must* have the ability to regularly and quickly engage senior leadership and subject matter experts.

Risk management *must* be formally integrated into program management and made indistinguishable from it.

A risk management plan *must* be written, with its practices and procedures consistent with the program's system engineering approaches.

Participants *must* be trained in the program's specific risk management practices and procedures.

Risk management execution *must* be shared amongst all stakeholders.

Risks *must* be identified, assessed, and reviewed continuously, not just prior to major program reviews.

Risk considerations *must* be the *central focus* of program reviews.

Risk management working groups and review boards *must* be rescheduled, instead of canceled, when conflicts arise with other program needs.

Risk mitigation plans *must* be developed, success criteria defined, and their implementation monitored relative to achieving success criteria outcomes.

Risks *must* be assigned only to staff or to organizations with authority to implement mitigation actions and obligate resources.

Risk management *must* never be outsourced.

Risks that extend beyond traditional impact dimensions of cost, schedule, and technical performance *must* be considered (e.g., programmatic, enterprise, cross-program/cross-portfolio, and social, political, economic impacts).

Technology maturity and its future readiness *must* be understood.

The adaptability of a program's technology to change in operational environments *must* be understood.

Risks *must* be written clearly using the *Condition-If-Then* protocol.

The nature and needs of the program *must* drive the design of the risk management process and its procedures, within which a risk management software tool or database conforms — not the other way around.

A risk management software tool or database *must* be maintained with current risk status information; preferably, employ visualization and database software that rapidly produces "dashboard-like" risk and risk mitigation status reports for management.

Index

D

E

F

G

H

I

K

L

M

N

O

P

Q

R

S

V

W

Z

Statistics

STATISTICS:
a series of TEXTBOOKS and MONOGRAPHS

Drawing from the author's many years of hands-on experience in the field, **Analytical Methods for Risk Management: A Systems Engineering Perspective** presents the foundation processes and analytical practices for identifying, analyzing, measuring, and managing risk in traditional systems, systems-of-systems, and enterprise systems.

Following an introduction to engineering risk management, the book covers the fundamental axioms and properties of probability as well as key aspects of decision analysis, such as preference theory and risk/utility functions. It concludes with a series of essays on major analytical topics, including how to identify, write, and represent risks; prioritize risks in terms of their potential impacts on a systems project; and monitor progress when mitigating a risk's potential adverse effects. The author also examines technical performance measures and how they can combine into an index to track an engineering system's overall performance risk. In addition, he discusses risk management in the context of engineering complex, large-scale enterprise systems.

This textbook and practical guide enable an understanding of which processes and analytical techniques are valid and how they are best applied to specific systems engineering environments. After reading this book, you will be on your way to managing risk on both traditional and advanced engineering systems.

Features

- Presents the processes and analytical practices for identifying, measuring, and managing risk as it arises in the engineering of systems
- Describes a supplier–provider risk analytic framework for enterprise systems engineered by capabilities-based development approaches
- Explores research areas for the development of advanced risk analysis and management protocols
- Discusses success factors for managing risk in the engineering of systems
- Includes end-of-chapter exercises and numerous examples to illustrate how technical principles apply to actual engineering systems projects

CRC Press
Taylor & Francis Group
an **informa** business
www.crcpress.com

6000 Broken Sound Parkway, NW
Suite 300, Boca Raton, FL 33487
270 Madison Avenue
New York, NY 10016
2 Park Square, Milton Park
Abingdon, Oxon OX14 4RN, UK

C6374

ISBN: 978-1-58488-637-2
90000
9 781584 886372